Name: ______________________________

Math 1120 **Fractions and Arithmetic** **Spring 2011**

In this worksheet, the goal is to perform some arithmetic with fractions and exponents.

1. A class consists of $\frac{2}{5}$ freshmen, $\frac{1}{4}$ sophomores, and $\frac{1}{10}$ juniors; the rest are seniors. What fraction of the class is seniors? (Explain briefly how you reached your answer.)

   Solution: The quantity

   $$\frac{2}{5}+\frac{1}{4}+\frac{1}{10}=\frac{8}{20}+\frac{5}{20}+\frac{2}{20}=\frac{15}{20}=\frac{3}{4}$$

   reflects the collection of students that are freshmen, sophomores, or juniors. The key idea here is that the quantity "1" represents the entire class, and that $3/4$ accounts for all but $1/4$ of students in that class. Thus, $1/4$ of the class is seniors.

2. Change each of the following fractions to mixed numbers.

   (a) $\dfrac{14}{5}$

   Solution:

   $$\frac{14}{5}=\frac{10}{5}+\frac{4}{5}=2+\frac{4}{5}=2\frac{4}{5}.$$

   (b) $-\dfrac{47}{8}$

   Solution:

   $$-\frac{47}{8}=-\frac{40}{8}-\frac{7}{8}=-5-\frac{7}{8}=-\left(5+\frac{7}{8}\right)=-5\frac{7}{8}.$$

3. Approximate the following sum.

   $$\frac{7}{15}+\frac{28}{9}+3\frac{9}{17}+\frac{23}{12}$$

   Solution: Notice that $\frac{7}{15}<\frac{7}{14}=\frac{1}{2}$. Also, we have $\frac{28}{9}>\frac{27}{9}=3$ and $3\frac{9}{17}=3+\frac{9}{17}>3+\frac{9}{18}=3+\frac{1}{2}$. Finally, $\frac{23}{12}<\frac{24}{12}=2$. Thus, we have approximately

   $$\frac{1}{2}+3+3+\frac{1}{2}+2=8+2\cdot\frac{1}{2}=9.$$

4. Approximate the following product.

$$5\frac{4}{5} \cdot 3\frac{1}{10}$$

Solution: This question can be answered in several ways. Probably the best approach is to get rid of the mixed numbers and work with improper fractions. We note that

$$5\frac{4}{5} \cdot 3\frac{1}{10} = \frac{29}{5} \cdot \frac{31}{10}.$$

Now, notice that $\frac{29}{5} < \frac{30}{5} = 6$ and $\frac{31}{10} > \frac{30}{10} = 3$. In other words, the product is approximately $6 \cdot 3 = 18$.

5. If a and b are rational numbers, with $a \neq 0$ and $b \neq 0$, and if m and n are integers, which of the following are true and which are false? Justify your answers.

General Idea: When you want to show that something is mathematically valid, you should try to make an argument to that effect. (This is a challenging thing to do, in general.) When you want to show that something is mathematically invalid, coming up with an example of why it is wrong is usually the best approach.

(a) $(a+b)^m = a^m + b^m$

Solution: This is FALSE!! For example, if $a = 2$, $b = 3$ and $m = 2$, then the left hand side is

$$(a+b)^m = (2+3)^2 = 5^2 = 25.$$

But, with the same values, the right hand side is found to be

$$a^m + b^m = 2^2 + 3^2 = 4 + 9 = 13.$$

Since the left hand side (25) is not equal to the right hand side (13), we have shown that this equality is false.

(b) $(ab)^0 = 1$

Solution: This is TRUE!! For any numbers a and b (nonzero), we know that the product ab is also a nonzero number. But, any nonzero number raised to the zero power is 1.

(c) $a^m \cdot a^m \cdot a^n = a^{2m+n}$

Solution: This is TRUE!! We will simplify the left hand side and eventually reach the right hand side:

$$a^m \cdot a^m \cdot a^n = (a^m \cdot a^m) \cdot a^n = (a^{m+m})a^n = a^{2m}a^n = a^{2m+n}.$$