I want my students to think and feel like mathematicians when I teach. For some, diving headfirst into a Calculus book comes naturally. For the more reluctant, however, my class may begin as little more than a degree requirement. This often leads to trading a deeper understanding of the material for a list of steps to be discarded after the exam. I am intent on avoiding this approach to learning. Every semester I collect more lesson plans, techniques, and strategies that bring out the mathematician in each student. When time constraints necessitate lecturing, I aim to make it engaging and accessible to everyone. But even better is to step back and let students take control of their own learning.

When trying to write a compelling lecture, I have found the following framework useful: $I \text{ will } = I \text{ can } + I \text{ want.}$ My style of lecture focuses first on the $I \text{ want}$ component—giving students a reason to care about the day’s lesson. This could be anything from a practical application to an interesting anecdote. Before studying center of mass in multivariable calculus, I present five scalene triangular boards with gridlines and a separate one-inch-thick dowel. I tell students that class will end with a group exercise on calculating their board’s center of mass, and each group will have one chance to balance it on the dowel. Of course most lectures do not have such theatrics. The divergence theorem begins with the motivating pain of calculating a surface integral over a cube, and finite series are introduced through the (probably heavily embellished) story of Gauss computing $1 + 2 + \cdots + 100$ instantly as a first grader. But even with a good hook, I have lost students who felt that the math was out of reach. This is the $I \text{ can}$ component, and in my experience, missing it will at best lead to the “can I just get a list of steps” mentality. My primary strategy is to tersely scaffold my lectures. If I am efficient enough with my words, a difficult learning objective can be built up from the basics without losing the interest of those with stronger backgrounds. In teaching Lagrange multipliers, a review of solving systems of linear equations sneaks in before anyone realizes we have switched from Calculus 3 to Algebra 1. My feeling, though, is that having to walk this fine line rather than fully differentiating my instruction is a good reason to avoid lecture in the first place.

With an unrushed curriculum I prefer a student-led approach. At CU Boulder this was especially possible with Calculus 1. As an example, before introducing the intermediate value theorem I write three true/false statements on the board: “At some point in your life, you weighed exactly 30 pounds,” “At some point in your life, your height in inches was equal to your weight in pounds,” and “There are always two points on opposite sides of the equator that have the same temperature.” Without knowing anything about the underlying mathematics, the class breaks into small groups to debate. My preference is to mix mid- with low-background students and mid- with high-background students. This structure encourages collaboration and increases the likelihood that everyone stays on the same page. It also allows me to devote more attention to groups that need it, while others are challenged by the intentionally difficult third statement. The theorem is finally introduced at the end of class, and everyone can then apply their understanding on the homework and solidify it with feedback. For me this exercise (and active learning in general) has proven to be a more effective teaching tool than rifling through the traditional list of practice problems on the board.

Keeping every student engaged can be challenging, but it has been helpful to have a community of instructors that share my enthusiasm for teaching here in the department. We learn from each other’s successes and failures in the classroom. The three statements I use for the intermediate value theorem are borrowed from a fellow teacher, and several have borrowed my center-of-mass boards. Developing new ideas and learning from others is part of what excites me about teaching. I look forward to continuing the never-ending improvement process with new colleagues.

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1 This was introduced to me as part of the Teach for America philosophy circa 2009.