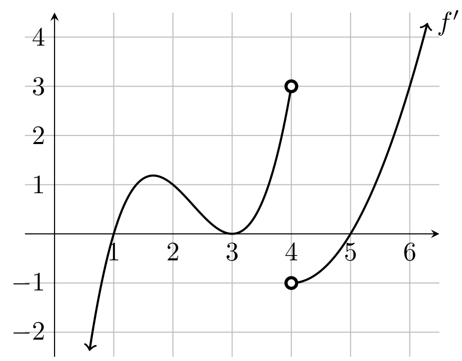
Name: ANSWER KEY Score: ________

1. Let $f(x)$ be a continuous function. The graph of its derivative, $f'(x)$, is shown below.



(a) (2 points) What are the critical points of f? Explain.

The critical numbers of f are the values of x where $f'(x) = 0$ or is undefined. The critical points are $x = 1, 3, 4, 5$.

(b) (1 point) Where does f have local minima? Explain.

By the first derivative test, since f' goes from negative to positive at $x = 1$ and $x = 5$, f has local minima at $x = 1$ and $x = 5$.

(c) (1 point) Where does f have local maxima? Explain.

By the first derivative test, since f' goes from positive to negative at $x = 4$, f has a local maximum at $x = 4$.

(d) (2 point) Where does f have inflection points? Explain.

The points where f' goes from increasing to decreasing or from decreasing to increasing are inflection points. The inflection points of f are at $x = 1.7$ and $x = 3$.

(e) (1 point) Is $f(1)$ larger than, smaller than, or equal to $f(3)$? Explain.

Since $f'(x)$ is positive on $(1, 3)$, f must be increasing between $x = 1$ and $x = 3$. Thus $f(3) > f(1)$.

2. (3 points) Suppose that we know that $f(x)$ is continuous and differentiable everywhere. Also suppose that $f(x)$ has two roots. Prove that $f'(x)$ must have at least one root.

   [Hint: Use the Mean Value Theorem.]

   Since $f(x)$ has two roots, we know that there exist values a and b such that $f(a) = 0$ and $f(b) = 0$. The slope of the secant line between $x = a$ and $x = b$ is

$$\frac{f(b) - f(a)}{b - a} = \frac{0}{b - a} = 0$$

   We know f is continuous and differentiable, so we may use the Mean Value Theorem. According to the MVT, there exists an x-value c between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

   But we already calculated that $\frac{f(b) - f(a)}{b - a} = 0$, so $f'(c) = 0$. This means $f'(x)$ has at least one root at $x = c$.