

3D PRINTED MANIPULATIVES FOR CALCULUS

The Project

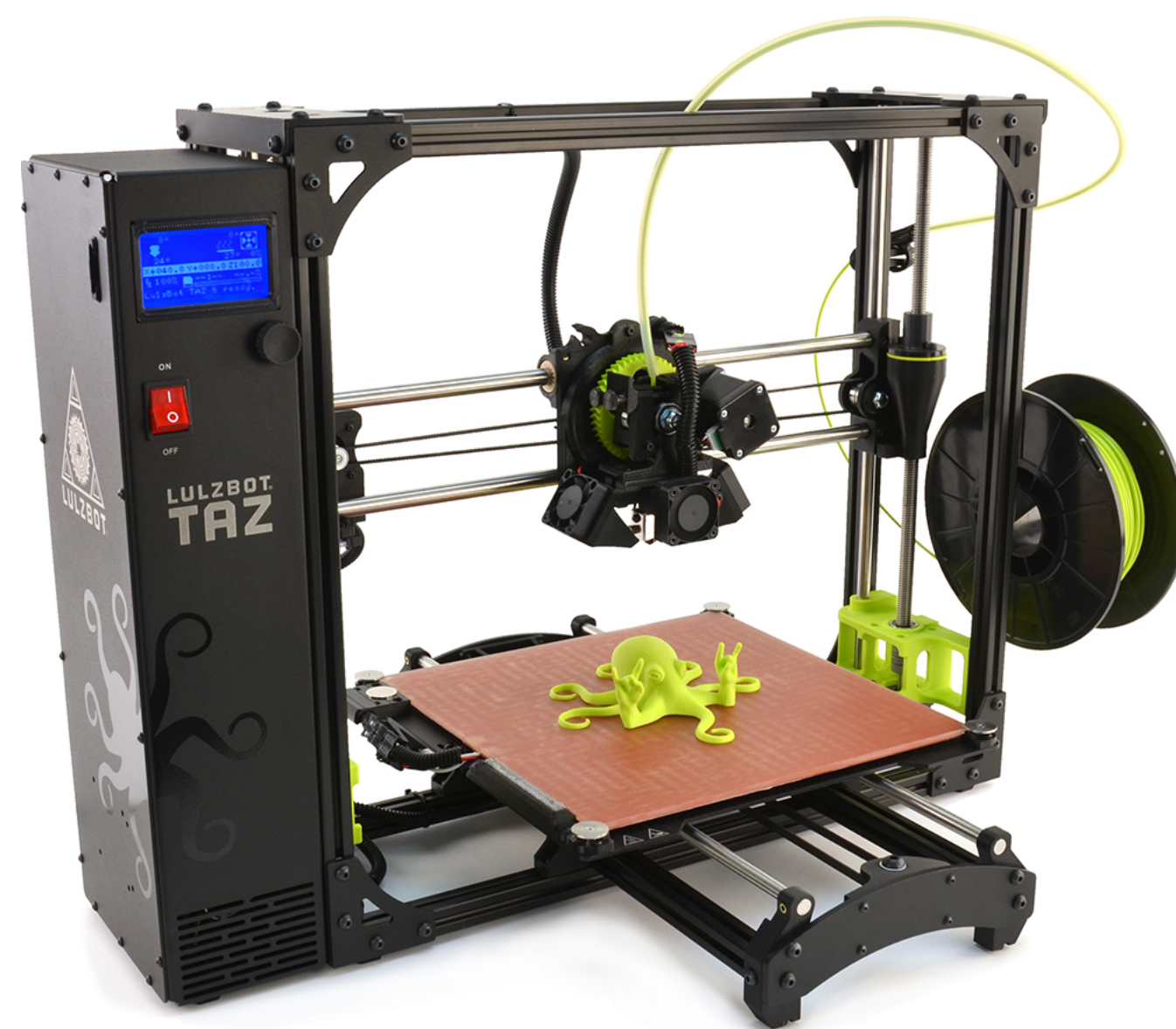
Calculus students can find it challenging to visualize shapes and objects in 3 dimensions. As calculus progresses from discussing curves and areas in 2 dimensions to volumes in 3 dimensions, students benefit from seeing physical models for these objects. We used a 3D printer to develop models and activities for our calculus sequence.

For more information about the models we have created and to find the source codes, visit our website at <https://cu3d.github.io/>.



Making the Models

3D PRINTER: We purchased the LulzBot TAZ 6 desktop printer for approximately \$2,500.



SOFTWARE: We use Python and Blender to create STL files of the models. Cura reads the STL files and interfaces with the printer. All of these programs are free and open-source.

SUPPLIES: We use mid-range quality plastic printing filament, which costs about \$25 per kg. Each 1 kg reel of filament can produce around 7-10 sets of activity models (see green models at right).

SUPPORT: The printer, supplies, and costs of development were paid for by the CU Math Department and a grant from Arts & Sciences Support of Education Though Technology (ASSETT) at CU Boulder.

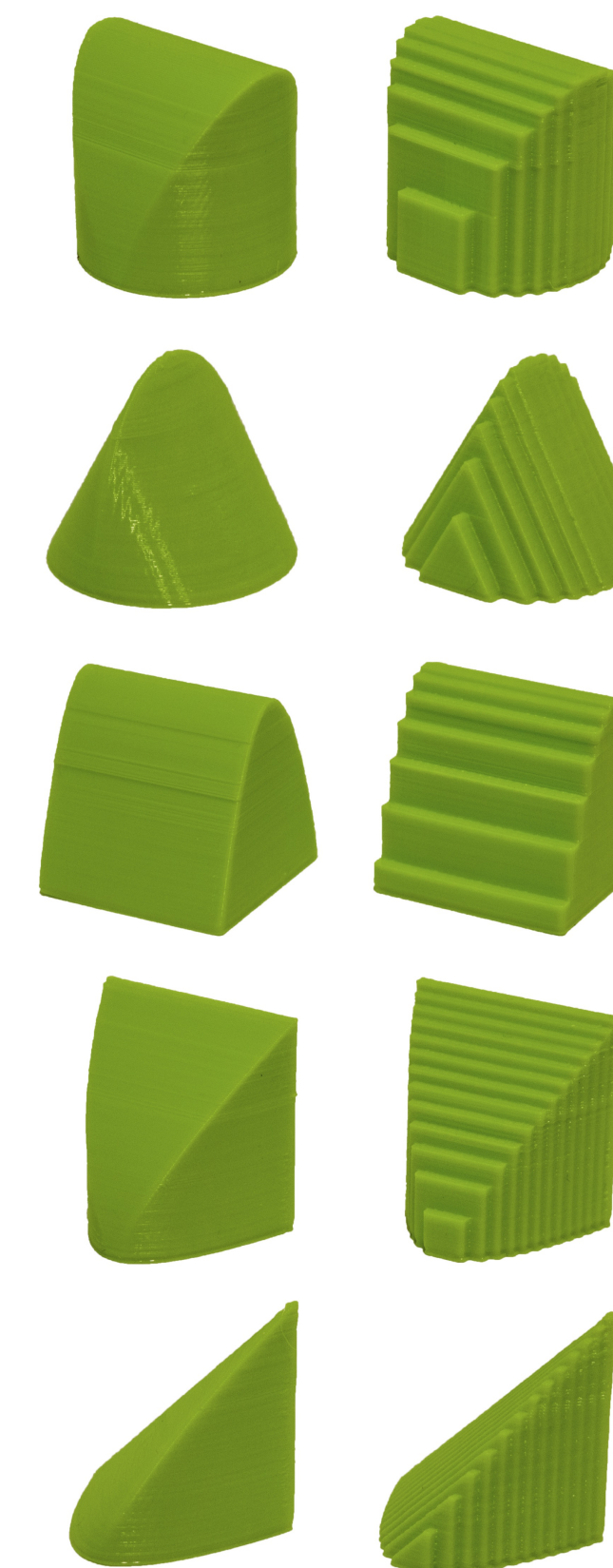
Calculus II Activity

TOPIC: Compute the volume of solids with known cross sections.

ACTIVITY: Students use the models to draw the base and a typical slice of each solid object. Then they use this information to write an integral that represents the volume of the solid.

Math 2300 Solids with Known Cross Sections

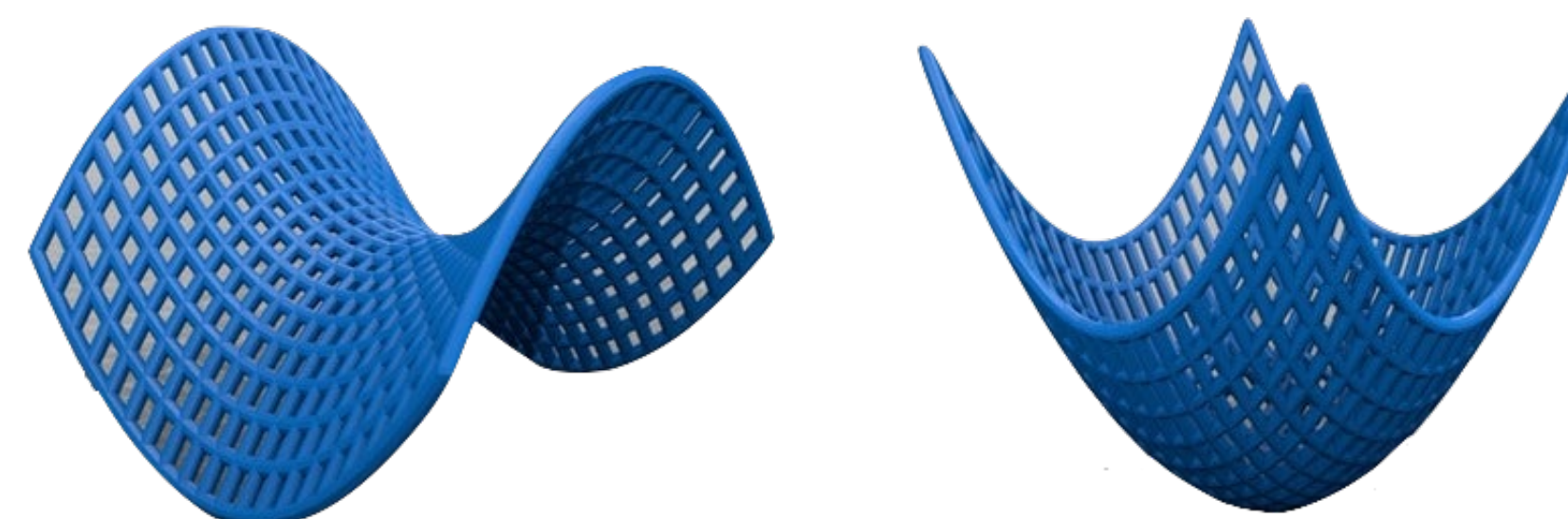
Written Description	Draw and label the base. Draw the bottom of one of the slices.	Draw one slice and label its dimensions.	Write the integral for the volume.
The base is a circle of radius 2 centered about the origin. The cross sections perpendicular to the x -axis are squares.			$\int_{-2}^2 (2\sqrt{4-x^2})^2 dx$
The base is a circle of radius 2 centered about the origin. The cross sections perpendicular to the x -axis are equilateral triangles.			$\int_{-2}^2 \frac{1}{2} (2\sqrt{4-x^2})(\sqrt{3}(4-x^2)) dx$
The base is a square with vertices at the points $(-2, -2)$, $(-2, 2)$, $(2, -2)$, and $(2, 2)$. The cross sections are rectangles of height $f(x) = -x^2 + 4$ and are perpendicular to the x -axis.			$\int_{-2}^2 4(-x^2+4) dx$
The base is the region enclosed by $y = x^2$ and $y = 3$. The cross sections perpendicular to the y -axis are squares.			$\int_0^3 (2\sqrt{3-y})^2 dy$
The base is the parabolic region $x = y^2$ and $x = 3$. The cross sections perpendicular to the x -axis are right isosceles triangles whose leg lies in the region.			$\int_0^3 \frac{1}{2} (2\sqrt{3-y^2})^2 dx$



The columns in the handout break down the steps for finding the integral representing the volume of each object. The smooth models and their Riemann sum approximations help the students visualize how the limit of the Riemann sum approximation gives the exact volume of the object.

Future Models

Surfaces for Calculus 3 can be difficult to draw or visualize. We plan to use a grid design to print sample surfaces like the ones below. From a pedagogical perspective, the grids demonstrate what a parametrization of a surface is.



Images: Thingiverse.com. "OpenSCAD Graph Generator by WClampitt1." *Thingiverse*, 30 Mar. 2017, www.thingiverse.com/thing:2214040. Licensed under the Creative Commons - Attribution license.

Other Models

DISCUSSION MODELS: We developed models to facilitate in-class discussions. The picture below shows a paraboloid (center) along with an approximation of the enclosed volume using disks (left) and shells (right). The pieces of the approximation models come apart to demonstrate the method of integration.



HOMEWORK MODELS: We created models to help students visualize the solids described in Calculus II homework problems. These models are available to students in their instructors' office hours as well as in the Mathematics Academic Resource Center at CU Boulder (free tutoring lab).

The example below is from *Single Variable Calculus, Concepts and Contexts, 4e* by James Stewart Section 6.2 Exercise 46.

A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.



The Team

This project is graduate student-led and is assisted by undergraduates and faculty.

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GRADUATE: Sebastian Bozlee, Caroline Matson, Cherry Ng, Athena Sparks; CU Boulder

FACULTY: Faan Tone Liu; University of Denver

We are very excited to see what the next group of students produce!

