Exercise 6.4.19

Linear Algebra
MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 6.4.19 from Lay [LLM16, §6.4]:

Exercise 6.4.19. Suppose $A = QR$, where Q is $m \times n$ and R is $n \times n$. Show that if the columns of A are linearly independent, then R must be invertible.

Solution. The columns of A are linearly independent if and only if $\ker(A) = \{\mathbf{0}\}$. It follows that $\ker(R) = \{\mathbf{0}\}$, since if $\mathbf{x} \in \ker(R)$, then $A\mathbf{x} = QR\mathbf{x} = Q\mathbf{0} = \mathbf{0}$, so that $\mathbf{x} \in \ker(A) = \{\mathbf{0}\}$. A square matrix is invertible if and only if its kernel is trivial; therefore R is invertible. □

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References

[LLM16] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Fifth edition, Pearson, 2016.

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