Exercise 5.3.27

Linear Algebra
MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 5.3.27 from Lay [LLM16, §5.3]:

Exercise 5.3.27. Show that if A is both diagonalizable and invertible, then so is A^{-1}.

Solution. The first observation is that a diagonal matrix

$$D = \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{pmatrix}$$

is invertible if and only if each of the d_i is non-zero (since $\det D = \prod d_i$), and if D is invertible, then

$$D^{-1} = \begin{pmatrix} 1/d_1 & & \\ & \ddots & \\ & & 1/d_n \end{pmatrix}$$

is diagonal.

Now assume that A is both diagonalizable and invertible. Then we know that A^{-1} is also invertible (with inverse A), so that we only need to show that A^{-1} is diagonalizable. To this end, since A is diagonalizable, there exists an invertible matrix S such that

$$S^{-1}AS = D = \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{pmatrix}$$

is diagonal. Taking the inverse of both sides of the equality $S^{-1}AS = D$, we have that $(S^{-1}AS)^{-1} = S^{-1}A^{-1}S = D^{-1}$ is diagonal (above we showed that D^{-1} was diagonal. Therefore, A^{-1} is diagonalizable, as well. $\square$

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References

[LLM16] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Fifth edition, Pearson, 2016.

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