

Exercise 4.1.6

Linear Algebra MATH 2130

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ABSTRACT. This is Exercise 4.1.6 from Lay [LLM16, §4.1]:

Exercise 4.1.6. Determine if the set of all polynomials

$$\{p(t) = a + t^2 : a \in \mathbb{R}\}$$

is a subspace of $\mathbb{R}[t]_n$ (“ \mathbb{P}_n ”), the vector space of real polynomials in t of degree at most n , for an appropriate value of n .

Solution. The set of all polynomials

$$\{p(t) = a + t^2 : a \in \mathbb{R}\}$$

is *not* a subspace of $\mathbb{R}[t]_n$ (“ \mathbb{P}_n ”) for any n . It is not even a subspace of $\mathbb{R}[t]$, the space of all polynomials.

Indeed, the set is not closed under addition. If we have $a + t^2$, and we add it to itself, we get $(a + t^2) + (a + t^2) = 2a + 2t^2$, which is not in the given set of polynomials since the coefficient of t^2 is not equal to 1. (Note that the observation that $2(a + t^2) = 2a + 2t^2$ also shows the set is not closed under scaling.) □

REFERENCES

[LLM16] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Fifth edition, Pearson, 2016.

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