Exercise 1.2.7

Linear Algebra
MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 1.2.7 from Lay [LLM16, §1.2]:

Exercise 1.2.7. Find the general solutions of the system whose augmented matrix is given below.

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

Solution. We start with the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right]$$

and then put the left hand side of the matrix in RREF. To do this, we first add -3 times the first row to the second row, giving

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{array}\right]$$

Dividing the second row by -5, we have

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Adding -4 times the second row to the first row, we have

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

The left hand side of the matrix above is in RREF.

Date: August 28, 2022.

Modifying the matrix above (with rows that are all zero except for one entry, which is a -1) until the left hand side is a square matrix with either 1 or -1 on the diagonal, we have

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & -5 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Therefore, the solutions are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} t + \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}, \quad t \in \mathbb{R}$$

□

Remark 0.1. One can alternatively (by setting $t = -x_2$) write the solutions as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}, \quad x_2 \in \mathbb{R}$$

which is the same as writing $x_1 = -3x_2 - 5$, x_2 is free, and $x_3 = 3$.

REFERENCES

[LLM16] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Fifth edition, Pearson, 2016.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: casa@math.colorado.edu