Exercise A.14

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise A.14 from Fraleigh [Fra03, Appendix: Matrix Algebra]:

Exercise A.14. Prove that if $A, B \in \mathrm{M}_n(\mathbb{C})$ are invertible, then AB and BA are invertible also.

Solution. Suppose that $A, B \in \mathrm{M}_n(\mathbb{C})$ are invertible. Then $(AB)(B^{-1}A^{-1}) = ABB^{-1}A^{-1} = AIA^{-1} = AA^{-1} = I$, and similarly, $(B^{-1}A^{-1})(AB) = B^{-1}B = I$, so that AB is invertible. We also have $(BA)(A^{-1}B^{-1}) = BB^{-1} = I$ and $(A^{-1}B^{-1})(BA) = A^{-1}A = I$, so that BA is invertible. $\square$

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REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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