Exercise 8.30

Abstract Algebra 1
MATH 3140

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ABSTRACT. This is Exercise 8.30 from Fraleigh [Fra03, §8]:

Exercise 8.30. Determine whether the map ("function")

$$f_1 : \mathbb{R} \to \mathbb{R}$$

defined by $f_1(x) = x + 1$ is a permutation of $\mathbb{R}$.

Solution. The map ("function") $f_1 : \mathbb{R} \to \mathbb{R}$ defined by $f_1(x) = x + 1$ is a permutation of $\mathbb{R}$; i.e., f_1 is a bijection (it is "both one-to-one and onto"). We have seen that this is equivalent to showing that f_1 has an inverse map ("function"); i.e., a map ("function") $f_1^{-1} : \mathbb{R} \to \mathbb{R}$ such that for all $x \in \mathbb{R}$ we have $f_1^{-1}(f_1(x)) = x$ and $f_1(f_1^{-1}(x)) = x$.

I claim the inverse map ("function") f_1^{-1} is given by $f_1^{-1}(x) = x - 1$. To see this we have

$$(f_1^{-1} \circ f_1)(x) = f_1^{-1}(x+1) = (x+1) - 1 = x.$$

Similarly, we have

$$(f_1 \circ f_1^{-1})(x) = f_1(x-1) = (x-1) + 1 = x.$$

□

Remark 0.1. We can also show that f_1 is a bijection (it is "both one-to-one and onto") directly. Indeed, f_1 is injective ("one-to-one") since if $f_1(a) = f_1(b)$, then $a + 1 = b + 1$, implying that $a = b$. And f_1 is surjective ("onto") since if $a \in \mathbb{R}$, then $f_1(a-1) = a$.

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REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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