# Exercise 13.2

## Abstract Algebra 1
## MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 13.2 from Fraleigh [Fra03, §13]:

**Exercise 13.2.** Determine whether the map $\phi : \mathbb{R} \to \mathbb{Z}$ given by $\phi(x) = \lfloor x \rfloor$ (i.e., the greatest integer that is less than or equal to $x$) is a homomorphism of groups.

*Solution.* The map $\phi : \mathbb{R} \to \mathbb{Z}$ given by $\phi(x) = \lfloor x \rfloor$ is *not* a homomorphism of groups. For instance, we have

$$\phi\left(\frac{1}{2}+\frac{1}{2}\right) = \phi(1) = \lfloor 1 \rfloor = 1 \neq 0 = 0+0 = \lfloor \frac{1}{2} \rfloor + \lfloor \frac{1}{2} \rfloor = \phi\left(\frac{1}{2}\right) + \phi\left(\frac{1}{2}\right).$$

□

*Remark* 0.1. Another way to do this problem would be to observe that $\phi^{-1}(0) = [0,1) \subseteq \mathbb{R}$ is not a subgroup (since for instance $\frac{1}{2} \in [0,1)$, but $\frac{1}{2}+\frac{1}{2} \notin [0,1)$).

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## REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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