# Exercise 10.3

## Abstract Algebra 1
## MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 10.3 from Fraleigh [Fra03, §10]:

**Exercise 10.3.** Find all cosets of the subgroup $\langle 2\rangle$ of $\mathbb{Z}_{12}$.

*Solution.* There are two cosets of the subgroup $\langle 2\rangle$ of $\mathbb{Z}_{12}$, namely,

$$\langle 2\rangle \quad \text{and} \quad 1+\langle 2\rangle.$$

To see this, observe that

$$0+\langle 2\rangle = \langle 2\rangle = \{0,2,4,6,8,10\}.$$
$$1+\langle 2\rangle = \{1,3,5,7,9,11\}.$$

These left cosets are distinct. By Lagrange's theorem, the number of left cosets of $\langle 2\rangle$ in $\mathbb{Z}_{12}$ is given by:

$$(\mathbb{Z}_{12} : \langle 2\rangle) = |\mathbb{Z}_{12}|/|\langle 2\rangle| = 12/6 = 2,$$

so $\langle 2\rangle$ and $1+\langle 2\rangle$ are the left only cosets. Since the group $\mathbb{Z}_{12}$ is abelian, every left coset is also a right coset, and so these are all the cosets. □

*Remark* 0.1. Alternatively, we can conclude that $0+\langle 2\rangle$ and $1+\langle 2\rangle$ are the only two cosets by observing that every element of $\mathbb{Z}_{12}$ is contained in one of these two left cosets, and again, since $\mathbb{Z}_{12}$ is abelian, every left coset is also a right coset.

*Date*: October 5, 2021.

## REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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