

HOMEWORK 1

MATH 2001

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ABSTRACT. This is the first homework assignment. The problems are from Hammack [Ham18, Ch. 1, §1.1]:

- **Chapter 1 Section 1**, Exercises: 2, 8, 18, 30, 38, 40.

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CHAPTER 1 SECTION 1.1

Ch.1, §1.1, Exercise 2. Write the following set by listing its elements between braces: $\{3x + 2 : x \in \mathbb{Z}\}$.

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Solution to Ch.1, §1.1, Exercise 2. I worked with the entire class on this
 1 \implies solution.¹

$$\{3x + 2 : x \in \mathbb{Z}\} = \{\dots, -7, -4, -1, 2, 5, 8, \dots\}.$$

□

Ch.1, §1.1, Exercise 8. Write the following set by listing its elements between braces: $\{x \in \mathbb{R} : x^3 + 5x^2 = -6x\}$.

Solution to Ch.1, §1.1, Exercise 8. This solution was provided by Professor Casalaina. For $x \in \mathbb{R}$, we have

$$\begin{aligned} x^3 + 5x^2 = -6x &\iff x^3 + 5x^2 + 6x = 0 \\ &\iff x(x^2 + 5x + 6) = 0 \\ &\iff x(x + 2)(x + 3) = 0 \\ &\iff x = 0, \text{ or } x = -2, \text{ or } x = -3. \end{aligned}$$

Therefore,

$$\{x \in \mathbb{R} : x^3 + 5x^2 = -6x\} = \{0, -2, -3\}.$$

□

Ch.1, §1.1, Exercise 18. Write the following set in set-builder notation:

$$\{0, 4, 16, 36, 64, 100, \dots\}$$

¹You are encouraged to work together on homework assignments. However, for each problem you must write your own solution, and, as I have done here, you must indicate with whom you worked, and you must cite any resources you used in solving the problem.

Solution to Ch.1, §1.1, Exercise 18. This solution was provided by Professor Casalaina. We note that for $n = 0, \dots, 5$, we have the following values for $(2n)^2$:

n	$(2n)^2$
0	0
1	4
2	16
3	36
4	64
5	100

Since this agrees with the list we were given, we may write

$$\{0, 4, 16, 36, 64, 100, \dots\} = \{(2n)^2 : n \in \mathbb{Z}_{\geq 0}\},$$

where $\mathbb{Z}_{\geq 0}$ is the set of integers that are greater than or equal to zero. □

Ch.1, §1.1, Exercise 30.

Solution to Ch.1, §1.1, Exercise 30. □

Ch.1, §1.1, Exercise 38.

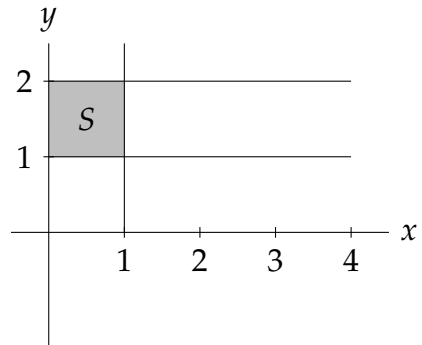
Solution to Ch.1, §1.1, Exercise 38. □

Ch.1, §1.1, Exercise 40. Sketch the following set of points in the x, y -plane:

$$S = \{(x, y) : x \in [0, 1], y \in [1, 2]\}$$

Solution to Ch.1, §1.1, Exercise 40. For this problem I first sketched my own solution by hand. However, to implement my solution in \LaTeX , I modified the tikz code from the webpage:

<https://tex.stackexchange.com/questions/140312/tikz-shading-region-bounded-by-s>



□

REFERENCES

[Ham18] Richard Hammack, *Book of Proof*, 3 ed., Creative Commons, 2018.

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