Math 2300-007: Quiz 15

Name: Solutions 5/2/18 Score: ________

1. (4 points) Find the tangent line to the polar curve $r = 1 + \sin(\theta)$ when $\theta = \frac{\pi}{3}$

- Need Slope $\left(\frac{dy}{dx}\Big|_{\pi/3} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\Big|_{\pi/3} \right)$ and Point $\left((x,y) \text{ when } \theta = \pi/3\right)$

- Parametric Representation:

$$\begin{cases} x = r\cos\theta = (1+\sin\theta)\cos\theta \\ y = r\sin\theta = (1+\sin\theta)\sin\theta \end{cases} \Rightarrow \begin{cases} \frac{dx}{d\theta} = (1+\sin\theta)(-\sin\theta) + \cos^2\theta \\ \frac{dy}{d\theta} = (1+\sin\theta)(\cos\theta) + (\cos\theta)(\sin\theta) \end{cases}$$

- Slope:

$$\frac{dy}{dx}\Big|_{\theta=\pi/3} = \frac{(1+\sin(\pi/3))\cos(\pi/3) + \cos(\pi/3)\sin(\pi/3)}{(1+\sin(\pi/3))(-\sin(\pi/3)) + \cos^2(\pi/3)} = \frac{(1+\frac{\sqrt{3}}{2})\frac{1}{2} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2}}{(1+\frac{\sqrt{3}}{2})\cdot\frac{-\sqrt{3}}{2} + (\frac{1}{2})^2} = \frac{\frac{1+\sqrt{3}}{2}}{\frac{-1-\sqrt{3}}{2}} = -1$$

- point:

$$(x,y)\Big|_{\theta=\pi/3} = \left((1+\sin(\pi/3))\cos(\pi/3),\ (1+\sin(\pi/3))\sin(\pi/3)\right) = \left((1+\tfrac{\sqrt{3}}{2})\tfrac{1}{2},\ (1+\tfrac{\sqrt{3}}{2})\cdot\tfrac{\sqrt{3}}{2}\right) = \left(\tfrac{1}{2}+\tfrac{\sqrt{3}}{4},\ \tfrac{\sqrt{3}}{2}+\tfrac{3}{4}\right)$$

- Tangent Line: $y - y_0 = m(x - x_0) \Rightarrow \boxed{y - \frac{\sqrt{3}}{2} - \frac{3}{4} = -1\cdot\left(x - \frac{1}{2} - \frac{\sqrt{3}}{4}\right)}$

2. (4 points) What is the arc length of the polar curve $r = \theta^2$ for $0 \leq \theta \leq 2\pi$.

$$\text{Arc Length} = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{2\pi} \sqrt{\theta^4 + 4\theta^2}\, d\theta = \int_0^{2\pi} \boxed{\sqrt{\theta^2}} \sqrt{\theta^2+4}\, d\theta$$

$$= \int_0^{2\pi} |\theta| \sqrt{\theta^2+4}\, d\theta = \int_0^{2\pi} \theta\sqrt{\theta^2+4}\, d\theta = \int_{u=4}^{u=4\pi^2+4} \frac{1}{2}\sqrt{u}\, du$$

$$\left\{ \begin{array}{l} u = \theta^2+4 \\ du = 2\theta\, d\theta \\ \frac{1}{2}du = \theta\, d\theta \end{array} \right\}$$

$$= \int_4^{4\pi^2+4} \frac{1}{2}u^{1/2}\, du = \frac{1}{3}u^{3/2}\Big|_4^{4\pi^2+4} = \boxed{\frac{1}{3}(4\pi^2+4)^{3/2} - \frac{1}{3}\cdot 8}$$

3. (2 points) During which of the following time intervals are you available for a review? Make an ‘x’ in the box for each time slot you can attend.

24 Responses

	Friday 5/4	Saturday 5/5	Sunday 5/6
10-12	9	8	12
12-2	8	13	13
2-4	13	16	19
4-6	15	14	17
6-8	13	14	15

Reviews are:

Saturday: 4-6 pm in Math 350

Sunday: 2-4 pm in Math 350