§8.7: Taylor's Inequality

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Key Points:

- **Goal:** Find the size of the error when using a Taylor Polynomial to estimate a function. The error (remainder) is $R_n(x) = f(x) - T_n(x)$, where $T_n(x)$ is the nth degree Taylor polynomial of f.
- The size of the error is influenced by the following:
  - the degree n of the Taylor Polynomial
  - the distance between x and the point a around which the Taylor Series is centered
  - the size of $\left|f^{(n+1)}(x)\right|$ in the interval between x and a.
- **Taylor's Inequality:** If $f^{(n+1)}$ is continuous and $\left|f^{(n+1}(x)\right| \leq M$ between x and a, then the remainder $R_n(x)$ satisfies the inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}$$

- Procedure: Find M and fill in a. Then $R_n(x)$, n and x interact. You'll typically be given two of them and have to find the third.

Examples:

1. Use the 6th degree Tayolor Polynomial for e^x, centered at $a = 0$ to estimate e. How accurate is your estimate guaranteed to be?

2. How accurate is the 3rd degree Taylor Polynomial centered at 0 guaranteed to be in estimating $\sin(9°)$.

3. Show that the Taylor Series for $\cos(x)$ centered at $a = 0$ converges to $\cos(x)$.

4. How large should n be to estimate e to within six decimal places (0.0000005) using a Taylor Polynomial centered at $x = 0$.

5. Use a 5th degree Taylor Polynomial to estimate $\ln(1.1)$. How accurate is your estimate?