

§8.4: Absolute Convergence and Ratio Test

(Thanks to Faan Tone Liu)

Key Points:

- If the series $\sum |a_n|$ converges, then the original series $\sum a_n$ converges:

In this case, we say the series $\sum a_n$ is **absolutely convergent** or **converges absolutely**. This is a new way to prove series convergence!

- Series that are convergent but are not absolutely convergent are called **conditionally convergent**. One example is

- The **Ratio Test** is another tool we can use. Calculate $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$.

(i) If $L < 1$, then $\sum a_n$ converges absolutely (and thus converges)

(ii) If $L > 1$, including $+\infty$, then $\sum a_n$ diverges

(iii) If $L = 1$, the ratio test tells you nothing. Try something else.

- Helpful graphic:

| | $\sum a_n$ converges | $\sum a_n$ diverges |
|------------------------|----------------------|---------------------|
| $\sum a_n $ converges | | |
| $\sum a_n $ diverges | | |

Examples:

1. Does $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ converge or diverge?

2. Does $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converge or diverge?

3. Does $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ converge absolutely, converge conditionally, or diverge?

4. Does $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converge absolutely, converge conditionally, or diverge?

5. Does $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{e^n}$ converge absolutely, converge conditionally, or diverge?

6. Does $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$ converge absolutely, converge conditionally, or diverge?