Daily Quiz

- Get into a group of FOUR PEOPLE. No more, no less.
- Go to Socrative.com
- Room Name: HONG5824
- Enter the last names of everyone (e.g. Smith, Sparks, Bozlee, Pierson)
- Complete the quiz.

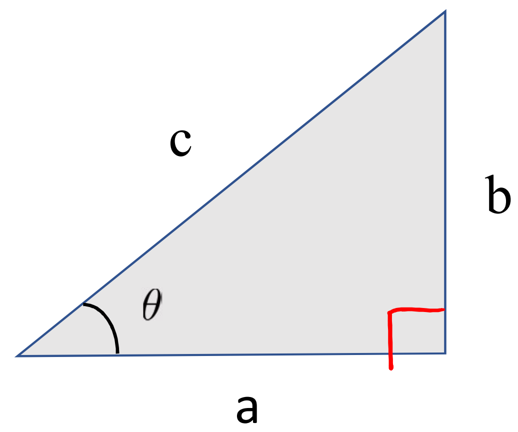
Trigonometric Identities Review

Right Angle Trigonometry

$\sin\theta = \frac{b}{c}$ $\csc\theta = \frac{c}{b}$

$\cos\theta = \frac{a}{c}$ $\sec\theta = \frac{c}{a}$

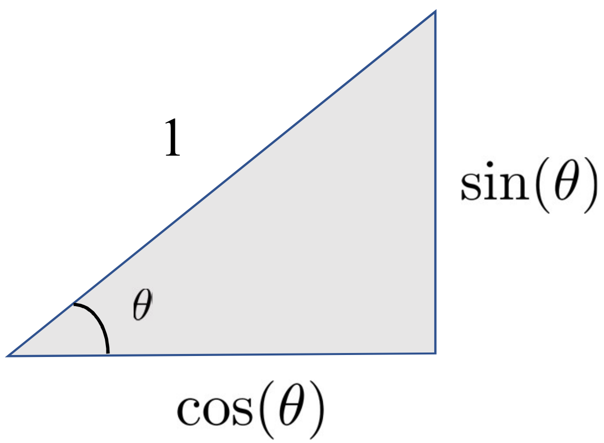
$\tan\theta = \frac{b}{a}$ $\cot\theta = \frac{a}{b}$



Trigonometric Identities Review

Pythagorean Theorem (3 identities)

1. $\sin^2(\theta) + \cos^2(\theta) = 1$
2. $1 + \tan^2(\theta) = \sec^2(\theta)$
3. $1 + \cot^2(\theta) = \csc^2(\theta)$



Trigonometric Identities Review

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

Half Angle Formulas

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$
$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

5.7 Trigonometric Integrals

An integral with an odd power of cos x Evaluate $\int \cos^3 x\, dx$.

$\cos^3 x = \cos^2 x \cdot \cos x$

$= (1 - \sin^2 x) \cos x$

$\int \cos^3 x\, dx = \int (1 - \sin^2 x) \cos x\, dx$

$u = \sin x$

$du = \cos x\, dx$

$\int (1 - \sin^2 x) \cos x\, dx$

$= \int (1 - u^2)\, du$

$\int (1 - u^2)\, du = u - \frac{u^3}{3} + C$

$= \sin x - \frac{\sin^3 x}{3} + C$

5.7 Trigonometric Integrals

An integral with an even power of sin x Evaluate $\int_0^{\pi} \sin^2 x \, dx$.

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\int_0^{\pi} \sin^2 x \, dx = \int_0^{\pi} \frac{1-\cos(2x)}{2} \, dx$$

$$= \int_0^{\pi} \left(\frac{1}{2} - \frac{\cos(2x)}{2}\right) dx$$

$$= \left[\frac{x}{2} - \frac{\sin(2x)}{4}\right]_0^{\pi}$$

$$= \left[\frac{\pi}{2} - \cancel{\frac{\sin(2\pi)}{4}}^{\,0}\right] - \left[\cancel{\frac{0}{2}}^{\,0} - \cancel{\frac{\sin(0)}{4}}^{\,0}\right]$$

$$= \frac{\pi}{2}.$$

5.7 Trigonometric Integrals

$\int \sin^3 x \cos^2 x \, dx$

$$\sin^3 x = \sin^2 x \cdot \sin x$$
$$= (1-\cos^2 x)\sin x$$

$$\int \sin^3 x \cos^2 x \, dx = \int (1-\cos^2 x)\sin x \cdot \cos^2 x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= \int (1-u^2)\, u^2 \,(-du)$$

$$= \int (u^4 - u^2)\, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$$

5.7 Trigonometric Integrals

$$\int \tan^3 x \sec x \, dx$$

$$\tan^2 x = \sec^2 x - 1$$

$$\int \tan^3 x \sec x \, dx = \int (\sec^2 x - 1) \tan x \sec x \, dx$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$\int (u^2 - 1) \, du = \frac{u^3}{3} - u + C$$

$$= \frac{\sec^3 x}{3} - \sec x + C$$

5.7 Trigonometric Integrals

$$\int_0^{\pi/4} \tan^2 x \sec^4 x \, dx$$

$\sec^2 x = 1 + \tan^2 x$

$$\int_0^{\pi/4} \tan^2 x \sec^4 x dx$$

$$= \int_0^{\pi/4} \tan^2 x \left(1 + \tan^2 x\right) \sec^2 x dx$$

$u = \tan x$

$du = \sec^2 x dx$

$u(\pi/4) = \tan \pi/4 = 1$

$u(0) = \tan 0 = 0$

$$= \int_{u(0)}^{u(\pi/4)} u^2 (1 + u^2) du$$

$$= \int_0^1 u^2 + u^4 \, du$$

$$= \left[\frac{u^3}{3} + \frac{u^5}{5}\right]_0^1$$

$$= \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

5.7 Trigonometric Integrals

$$\int \sec^3(x)\, dx$$

$u = \sec(x)$	$v = \tan x$
$du = \sec(x)\tan(x)dx$	$dv = \sec^2 x\, dx$

$$uv - \int v\,du = \sec(x)\tan(x) - \int \tan^2(x)\sec(x)dx = \int \sec^3(x)\,dx$$

Observe that

$$\int \sec^3(x)dx = \int (1+\tan^2(x))\sec(x)dx$$

$$= \int \sec(x)dx + \int \tan^2(x)\sec(x)dx. \quad \text{So}$$

$$\int \sec(x)dx + \int \tan^2(x)\sec(x)dx = \sec(x)\tan(x) - \int \tan^2(x)\sec(x)dx$$

5.7 Trigonometric Integrals

$$2\int \tan^2(x)\sec(x)\,dx = \sec(x)\tan(x) - \int \sec(x)\,dx$$

$$\int \sec(x)\,dx = \int \frac{\sec(x)\left[\sec(x)+\tan(x)\right]}{\left[\sec(x)+\tan(x)\right]}\,dx$$

$u = \sec(x) + \tan(x)$

$du = \left[\sec(x)\tan(x) + \sec^2(x)\right]dx$

$$\int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{du}{u} = \ln|u|$$

$$= \ln|\sec(x) + \tan(x)|$$

5.7 Trigonometric Integrals

Hence

$$2\int \tan^2(x)\sec(x)\,dx = \sec(x)\tan(x) - \ln|\sec(x)+\tan(x)|$$

and

$$\int \sec^3(x)\,dx = \int \sec(x)\,dx + \int \tan^2(x)\sec(x)\,dx$$

$$= \ln|\sec(x)+\tan(x)| + \frac{1}{2}\Big[\sec(x)\tan(x) - \ln|\sec(x)+\tan(x)|\Big] + C$$

$$= \frac{1}{2}\Big[\sec(x)\tan(x) + \ln|\sec(x)+\tan(x)|\Big] + C.$$