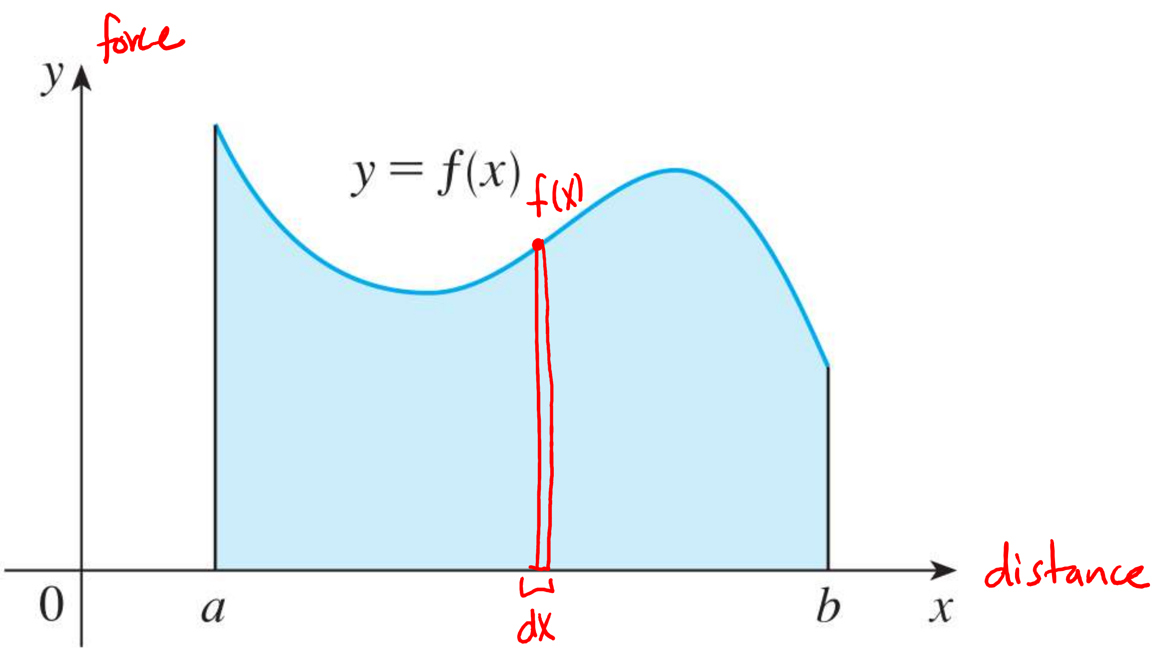
Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

6.6 Work

kg is a unit of mass
lb is a unit of force (weight)

- Work = Force x Distance.
- If force is a function that changes with respect to distance, then work can be thought of as the area under the curve.



$$W = \int_a^b \underbrace{f(x)}_{\text{force}} \, \underbrace{dx}_{\text{small distance}}$$

6.6 Work

When a particle is located a distance x feet from the origin, a force of $x^2 + 2x$ pounds acts on it. How much work is done in moving it from $x = 1$ to $x = 3$?

Force: $f(x) = x^2 + 2x$

work done $= \int_1^3 f(x)\,dx$

$= \int_1^3 (x^2 + 2x)\,dx$

$= \left[\frac{x^3}{3} + x^2\right]_1^3$

$= (9+9) - (\frac{1}{3} + 1)$

$= \frac{50}{3}$ ft-lb

6.6 Work

Hooke's Law states that the force required to maintain a spring stretched x units beyond its natural length is proportional to x:

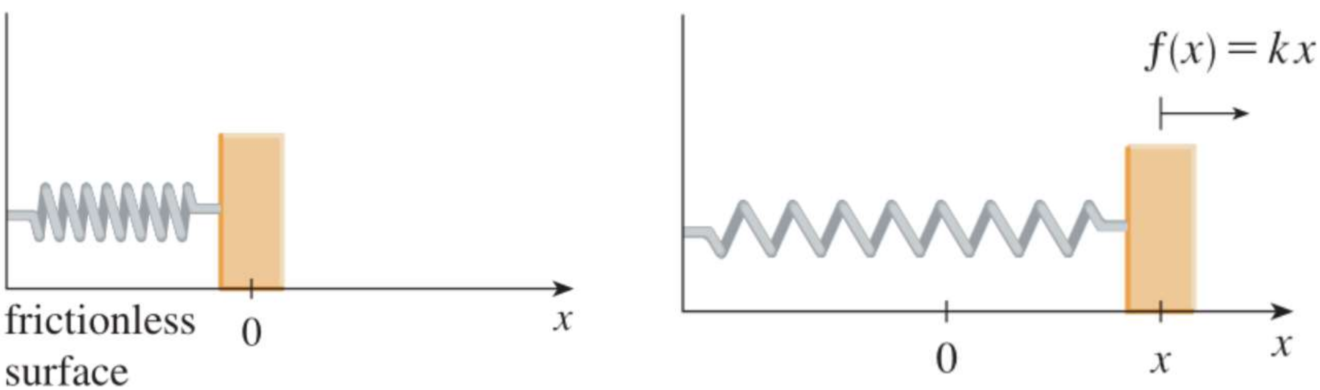
$$f(x) = kx$$

where k is a positive constant called the **spring constant**.

If the spring's natural location is not at the origin, then the equation becomes

$$f(x) = k(x - x_0)$$

where x_0 is the natural location of the spring.



(a) Natural position of spring

(b) Stretched position of spring

6.6 Work

A force of 40 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

$$\text{Force} = k(x - x_0)$$

$$N = \frac{\text{kg m}}{\text{sec}^2}$$

convert cm to m

$$40 = k(0.15 - 0.1)$$

$$800 = k$$

$$\text{work} = \int_{0.15}^{0.18} k(x - x_0)\,dx$$

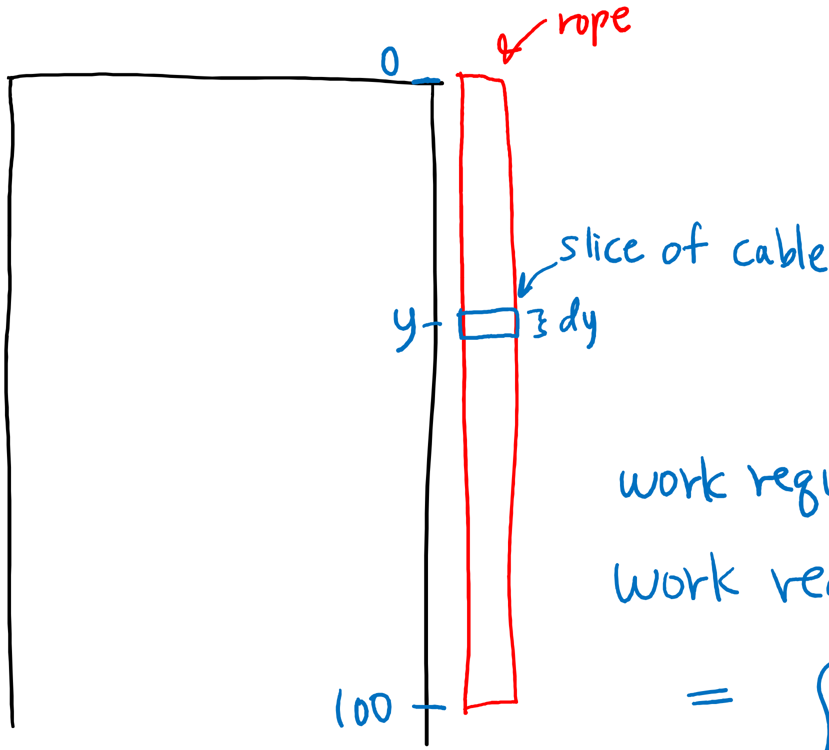
$$= \int_{0.15}^{0.18} 800(x - 0.1)\,dx$$

$$= 800\left[\frac{x^2}{2} - 0.1x\right]_{0.15}^{0.18}$$

$$= 1.56 \text{ Joules}$$

6.6 Work (Choosing a coordinate system)

A 200-lb cable is 100 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?



linear density $= \dfrac{200 \text{ lb}}{100 \text{ ft}} = 2 \dfrac{\text{lb}}{\text{ft}}$

weight of slice = linear density $\cdot$ length of slice

$= 2\,dy$

distance of the slice from the roof $= y$

work required to bring the slice to the roof $= (2dy)\,y$

work required to pull the cable to the roof

$$= \int_0^{100} \text{force} \cdot \text{distance} = \int_0^{100} 2y\,dy = \left[y^2\right]_0^{100}$$

$$= 10000 \text{ ft-lb}$$

6.6 Work (Choosing a coordinate system)

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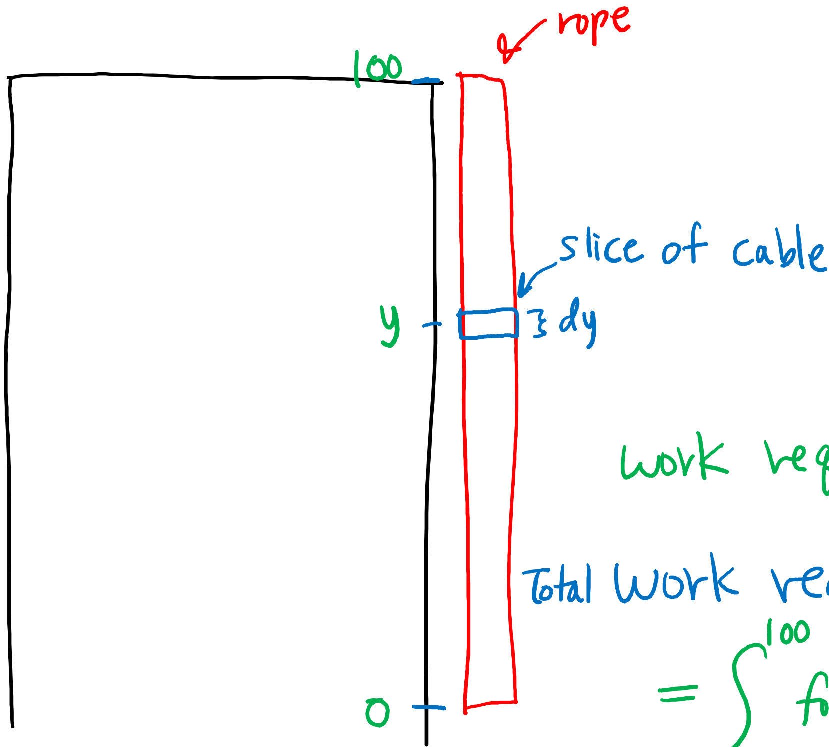


$$\text{linear density} = \frac{200 \text{ lb}}{100 \text{ ft}} = 2 \, \frac{\text{lb}}{\text{ft}}$$

$$\text{weight of slice} = \text{linear density} \cdot \text{length of slice} = 2\,dy$$

distance of the slice from the roof $= 100 - y$

work required to bring the slice to the roof $= (2\,dy)(100-y)$

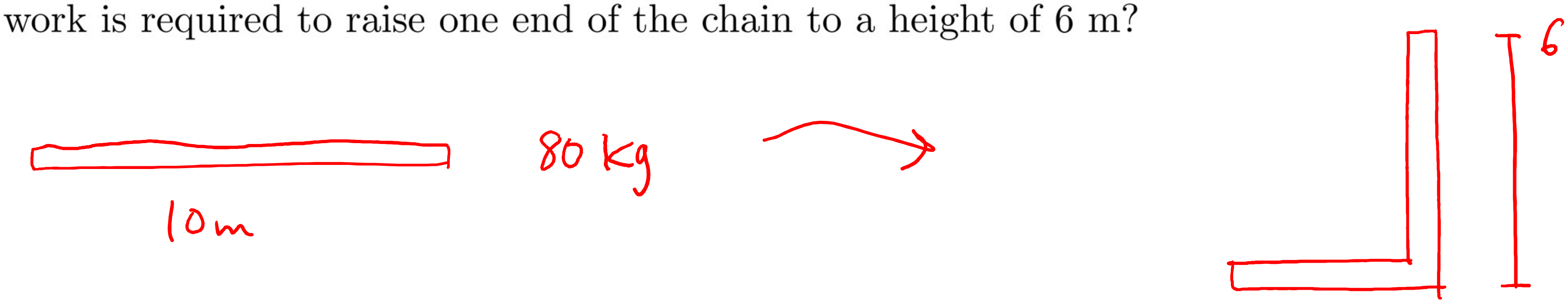
Total work required to pull the cable to the roof

$$= \int_0^{100} \text{force} \cdot \text{distance} = \int_0^{100} 2\,dy\,(100-y) = \int_0^{100} (200-2y)\,dy$$

$$= \left[200y - y^2\right]_0^{100} = 20000 - 10000 = 10000 \text{ ft-lb}$$

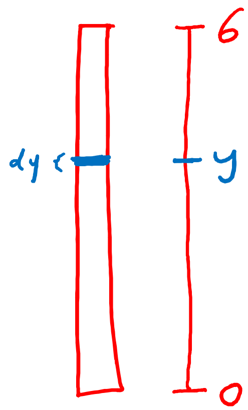
6.6 Work

A chain lying on the ground is 10 m long and its mass is 80 kg. How much work is required to raise one end of the chain to a height of 6 m?



Observation: Due to conservation of energy, the potential energy of the chain when it's raised is equal to the amount of work needed to move it to a height of 6 m.

Therefore it is enough to compute the potential energy of the raised chain. No work is done on the remaining chain on the ground so we can ignore it.



linear density of chain $= \dfrac{80\,\text{kg}}{10\,\text{m}} = 8\,\dfrac{\text{kg}}{\text{m}}$

mass of slice $= 8\,dy$

weight of slice $= (8\,dy)g = 8g\,dy$

distance the slice traveled $= y$

work done on the slice $= (8g\,dy)\,y$

$$\text{Total work} = \int_0^6 8g\,y\,dy = 8g\int_0^6 y\,dy = 8g\left[\frac{y^2}{2}\right]_0^6 = 144g \text{ Joules}$$

6.6 Work

Force = mass · acceleration

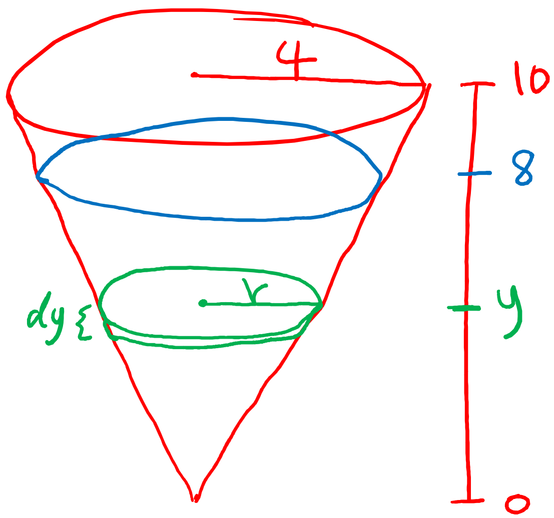
weight = gravitational force

= mg

(Gravitational constant)

$g = 9.81\ m/s^2$

A tank has the shape of an inverted circular cone with height 10 m and base radius 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m^3.)



Area of the slice $= \pi r^2$ $\quad m^2$

volume of the slice $= \pi r^2 dy$ $\quad m^3$

density of water $= 1000$ kg/m^3

weight of the slice $= 1000(\pi r^2 dy)\, g$

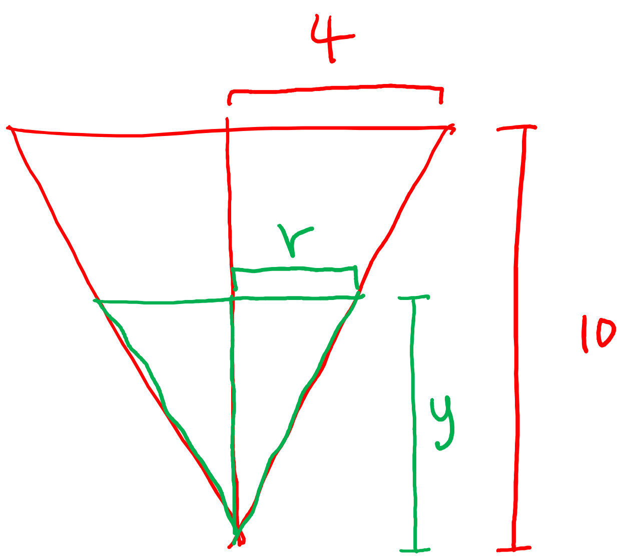
distance of the slice from the top $= 10 - y$

work required to bring the slice to the top

$= 1000(\pi r^2 dy)\, g\, (10-y)$

Total work required $= \int_0^8 1000(\pi r^2 dy)\, g\,(10-y)$

we need to solve for r. Observe that from the side, we have two similar triangles.



$$\frac{4}{10} = \frac{r}{y}$$

$$\boxed{r = \frac{2y}{5}}$$

we use this r for the integral above.

$$\text{Total work} = 1000\pi g \int_0^8 \left(\frac{2y}{5}\right)^2 (10-y)\,dy$$

$$= 1000\pi g \, \frac{4}{25} \int_0^8 y^2(10-y)\,dy$$

$$= 160\pi g \int_0^8 (10y^2 - y^3)\,dy$$

$$= 160\pi g \left[\frac{10y^3}{3} - \frac{y^4}{4}\right]_0^8$$

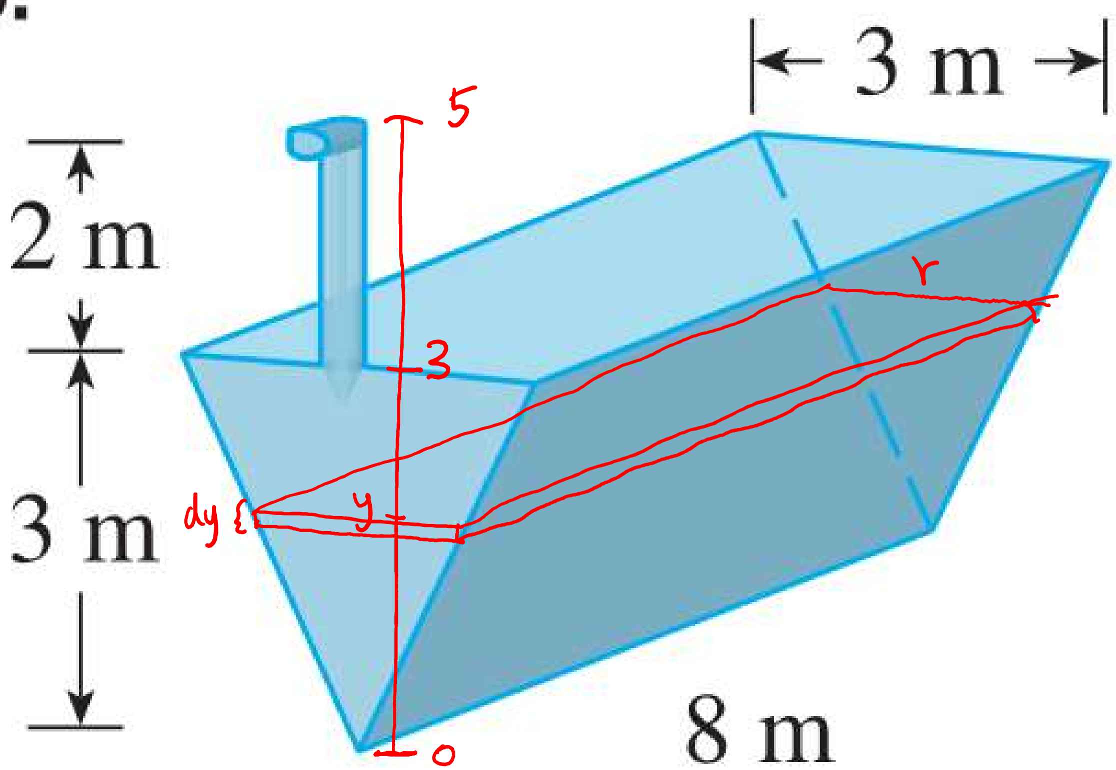
$$= 160\pi g \, \frac{2048}{3} \quad \text{Joules}$$

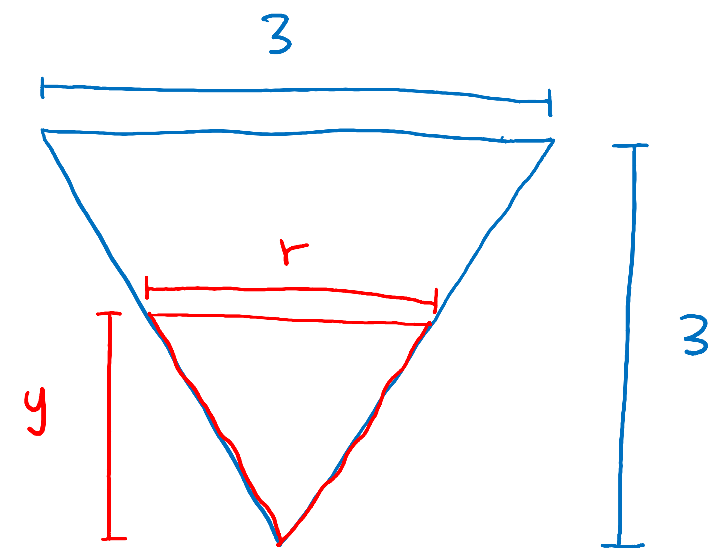
6.6 Work

A tank is full of water. Find the work required to pump the water out of the spout.

Similar Triangles

19.





$$\frac{3}{3} = \frac{r}{y}$$

$$r = y$$

volume of slice $= 8 r\,dy = 8y\,dy$

weight of slice $= (8y\,dy)\,1000\,g$

distance of the slice from the spout $= 5-y$

work required to bring the slice to the spout $= 8000g\,(5-y)\,y\,dy$

Total work $= \int_0^3 8000g\,(5-y)\,y\,dy$

$= 8000g \int_0^3 (5y-y^2)\,dy$

$= 8000g \left[\frac{5y^2}{2} - \frac{y^3}{3}\right]_0^3$

$= 8000g\left[\left(\frac{45}{2} - \frac{27}{3}\right) - 0\right]$

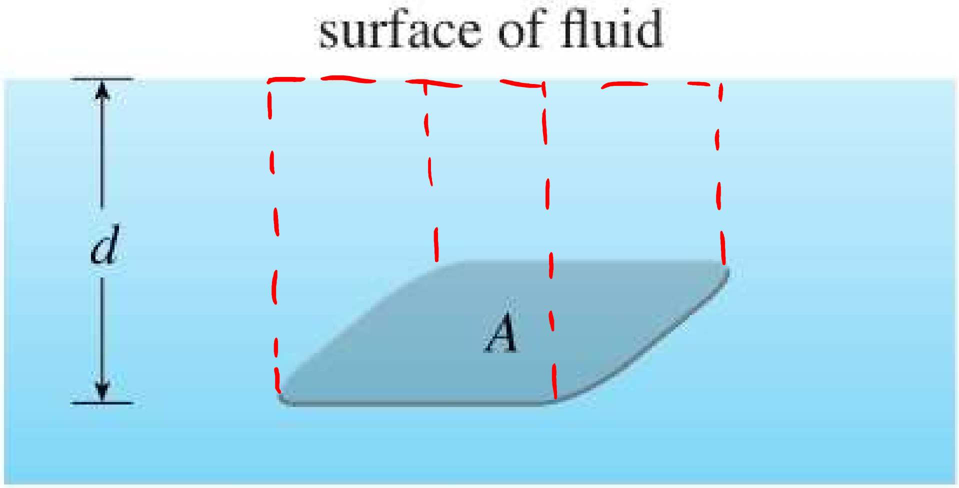
$= 8000g\,\frac{27}{2}$

$= 108000\,g$ Joules

6.6 Hydrostatic Pressure and Force

- Why do you feel more pressure as you dive deeper?
- Answer: There’s more water above you weighing you down as you dive deeper.

6.6 Hydrostatic Pressure and Force



Force?

Volume of the dotted box $= A \cdot d$

mass of the --- $=$ volume $\cdot$ density

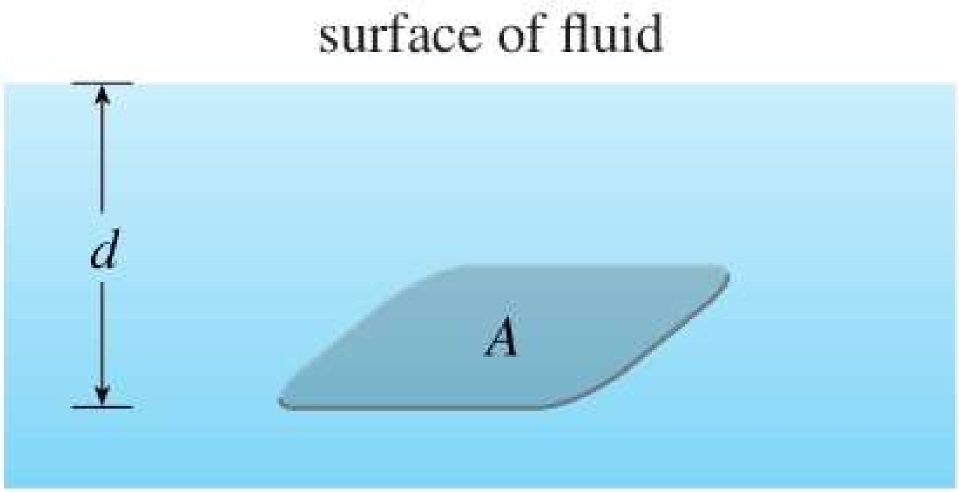
$$m = A \cdot d \cdot \rho$$

weight $= mg = A \cdot d \cdot \rho \cdot g =$ Force

pressure $= \dfrac{\text{Force}}{\text{Area}}$

pressure $= \dfrac{A \cdot d \cdot \rho \cdot g}{A} = \rho g d$

6.6 Hydrostatic Pressure and Force



$$P = \frac{F}{A} = \rho g d$$

density of water

gravity

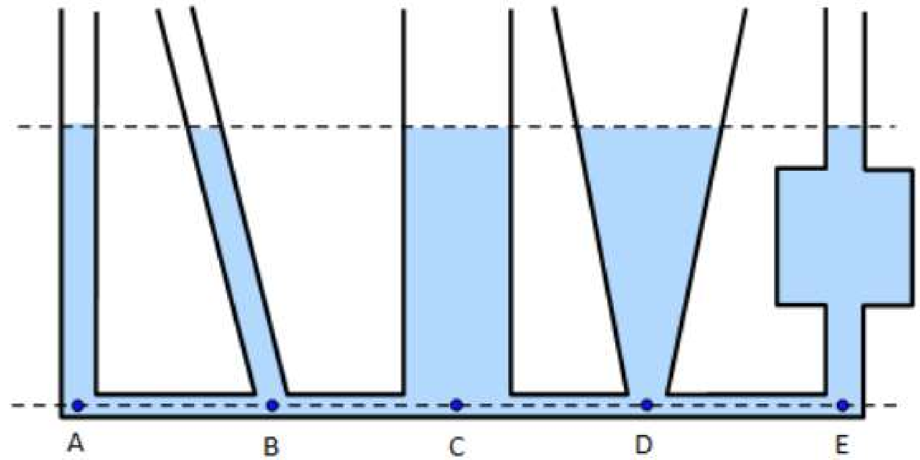
depth

$\rho = 1000 \text{kg}/m^3$

or $1 g/mL$

6.6 Hydrostatic Pressure and Force

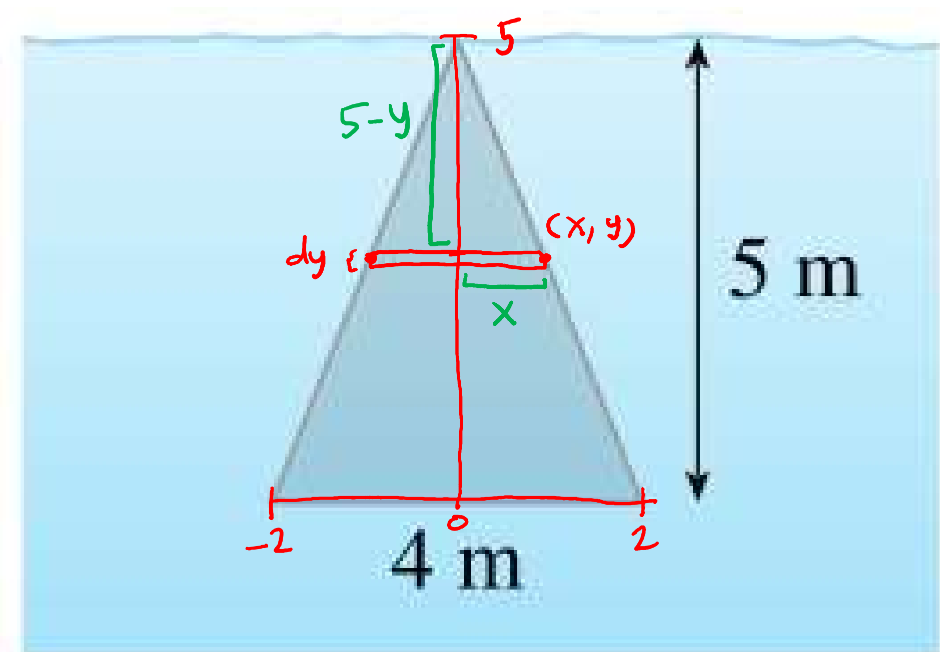
At any point in a liquid the pressure is the same in ALL directions. The pressure of a liquid is the same at any given depth below the surface regardless of the shape of the container.



Note: If the water level was different for each shape, then the bottom points would have different pressures.

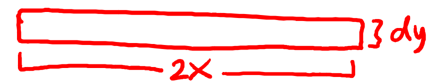
6.6 Hydrostatic Pressure and Force

A triangle with base 4 m and height 5 m is submerged vertically in water so that the tip is even with the surface. Express the hydrostatic force against one side of the plate as an integral.



$P = \rho g d$

$F = P A$



Area of slice $= 2x\,dy$

pressure on the slice $= \rho g (5-y)$

Force on the slice $= \rho g (5-y)\, 2x\, dy$

Total force

$$= \int_0^5 \rho g (5-y)\, 2x\, dy$$

Now we solve for x in terms of y.

Using similar triangles,

$$\frac{2}{5} = \frac{x}{5-y}$$

$$x = \frac{2(5-y)}{5}$$

So the total force is

$$\int_0^5 \rho g(5-y)\, 2\left(\frac{2(5-y)}{5}\right) dy \quad \text{Newton}$$