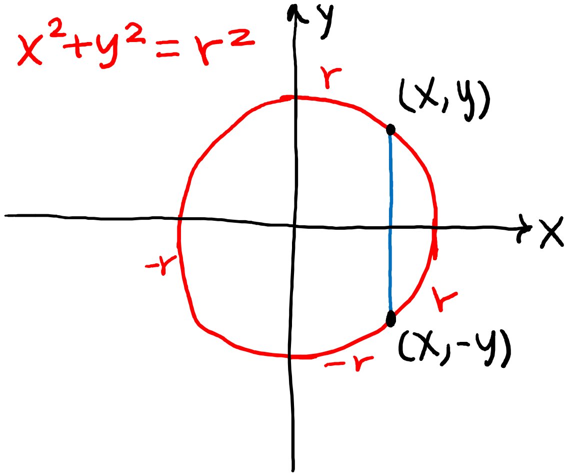
Daily Quiz

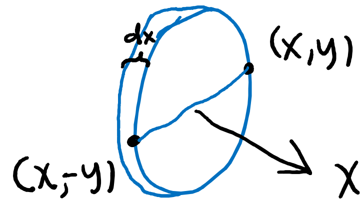
- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

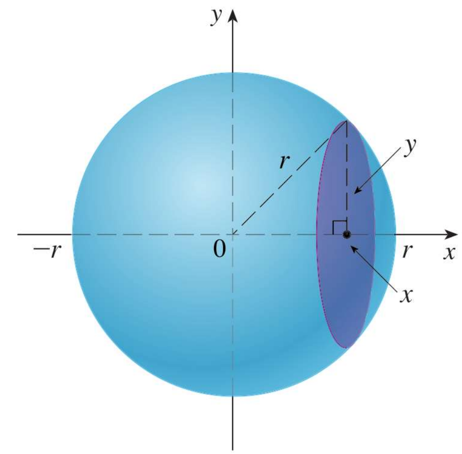
6.3 Solids of Revolution

Show that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.



cross-section is a disk





radius $= y$

Area $= \pi y^2$

Volume of slice $= \pi y^2 dx$

$$\text{Total volume} = \int_{-r}^{r} \text{volume of slice} = \int_{-r}^{r} \pi y^2 dx = \pi \int_{-r}^{r} (r^2 - x^2)\, dx$$

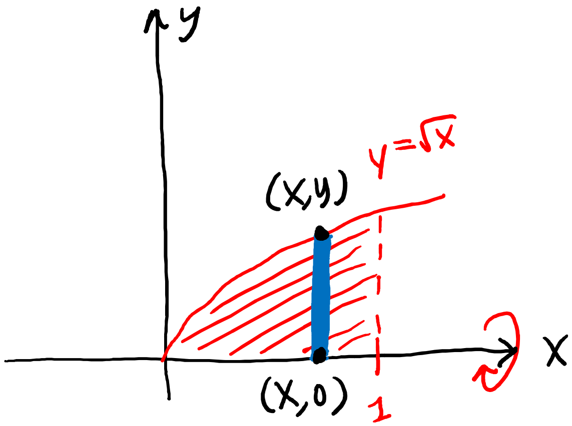
$$= \pi \left[r^2 x - \frac{x^3}{3} \right]_{-r}^{r}$$

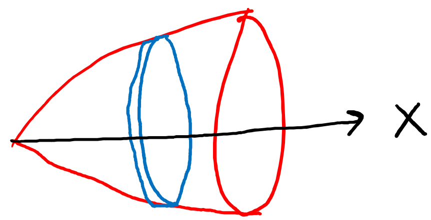
$$= \pi \left[\left(r^3 - \frac{r^3}{3} \right) - \left(-r^3 + \frac{r^3}{3} \right) \right]$$

$$= \frac{4\pi r^3}{3}$$

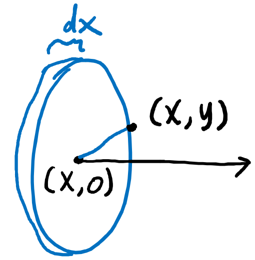
6.2 Rotating about the x-axis

Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.





cross-section is a disk



radius = y

Area = πy^2

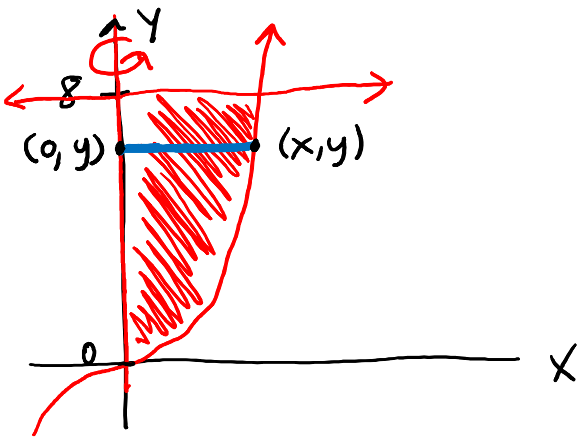
volume of slice = $\pi y^2 \, dx$

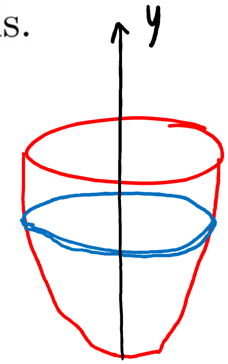
Total volume $= \int_0^1 \pi y^2 \, dx$

$$= \int_0^1 \pi (\sqrt{x})^2 \, dx = \int_0^1 \pi x \, dx = \pi \left[\frac{x^2}{2} \right]_0^1 = \frac{\pi}{2}$$

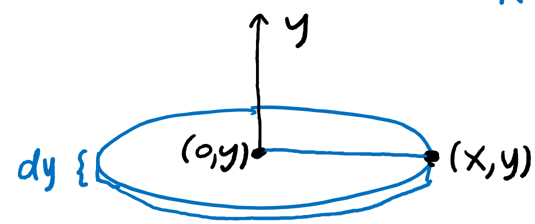
6.2 Rotating about the y-axis

Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.





cross-section is a disk



radius $= x$

Area $= \pi x^2$

volume of slice $= \pi x^2 dy$

$y = x^3$

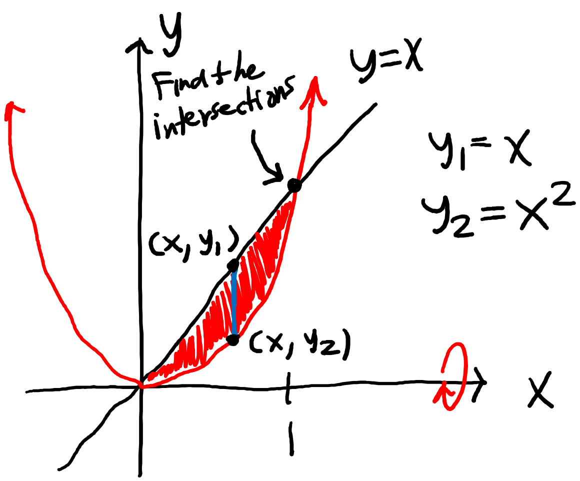
$y^{\frac{1}{3}} = x$

$$\text{Total volume} = \int_0^8 \pi x^2 dy = \int_0^8 \pi (y^{\frac{1}{3}})^2 dy$$

$$= \pi \int_0^8 y^{2/3} dy = \pi \left[\frac{3y^{5/3}}{5} \right]_0^8 = \frac{3\pi}{5} \left[8^{5/3} - 0 \right] = \frac{96\pi}{5}$$

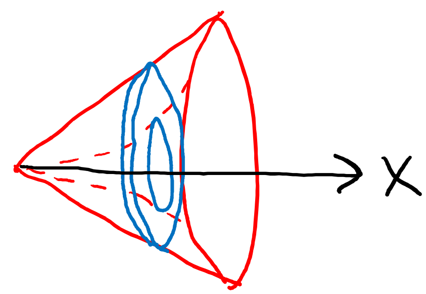
6.2 Cross-section is annulus

The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x-axis. Find the volume of the resulting solid.

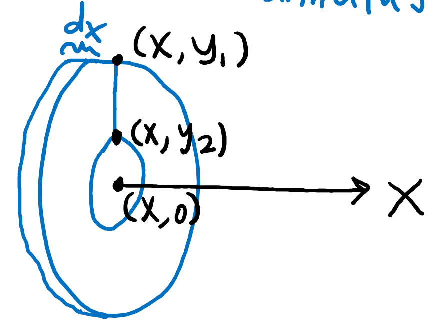


$y_1 = x$

$y_2 = x^2$



cross-section is an annulus



big radius $= R = y_1 - 0 = y_1$

small radius $= r = y_2 - 0 = y_2$

Area $= \pi R^2 - \pi r^2$

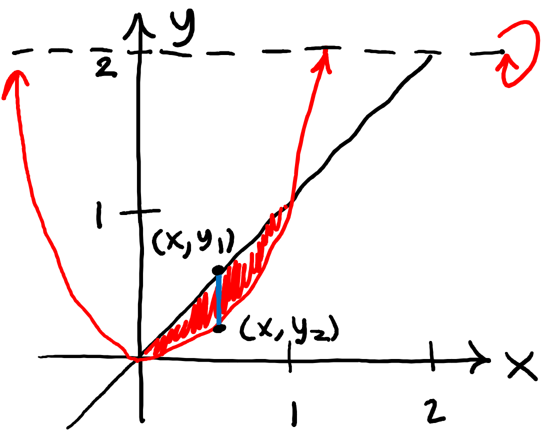
volume of slice $= (\pi R^2 - \pi r^2)\,dx$

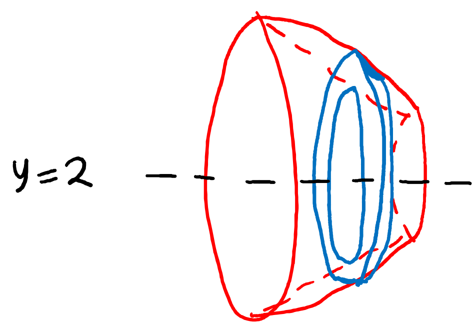
Total volume $= \displaystyle\int_0^1 (\pi R^2 - \pi r^2)\,dx$

$$= \pi \int_0^1 (y_1^2 - y_2^2)\,dx = \pi \int_0^1 (x^2 - (x^2)^2)\,dx = \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_0^1 = \frac{2\pi}{15}$$

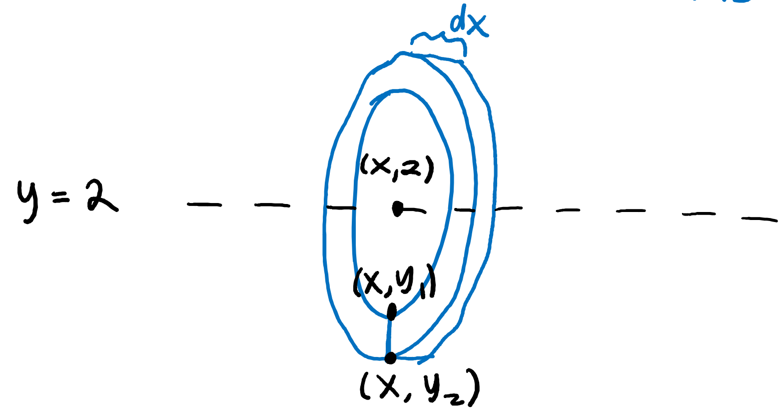
6.2 Cross-section is annulus

The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the line $y = 2$. Find the volume of the resulting solid.





cross-section is an annulus



$R = 2 - y_2 \quad r = 2 - y_1$

Area $= \pi R^2 - \pi r^2$

volume of slice $= (\pi R^2 - \pi r^2)\,dx$

$$\text{Total volume} = \int_0^1 (\pi R^2 - \pi r^2)\,dx$$

$$= \pi \int_0^1 (2-y_2)^2 - (2-y_1)^2\,dx$$

$$= \pi \int_0^1 (2-x^2)^2 - (2-x)^2\,dx$$

$$= \pi \int_0^1 \left[4 - 4x^2 + x^4 - (4 - 4x + x^2)\right] dx$$

$$= \pi \int_0^1 4x - 5x^2 + x^4 \, dx$$

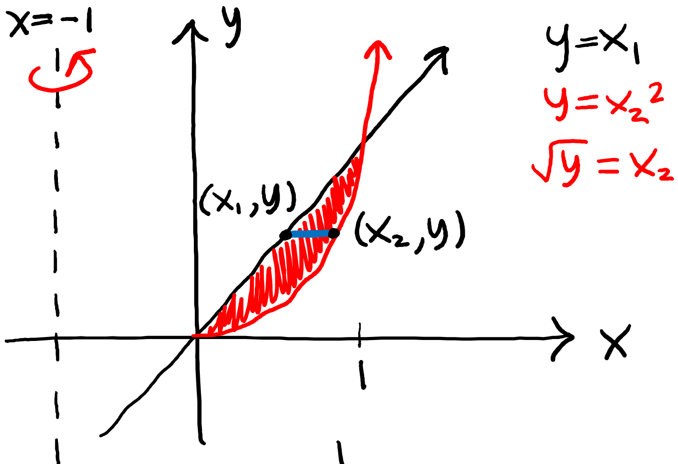
$$= \pi \left[2x^2 - \frac{5x^3}{3} + \frac{x^5}{5}\right]_0^1$$

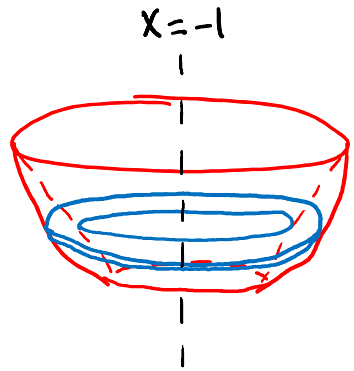
$$= \pi \left[\left(2 - \frac{5}{3} + \frac{1}{5}\right) - 0\right]$$

$$= \frac{8\pi}{15}$$

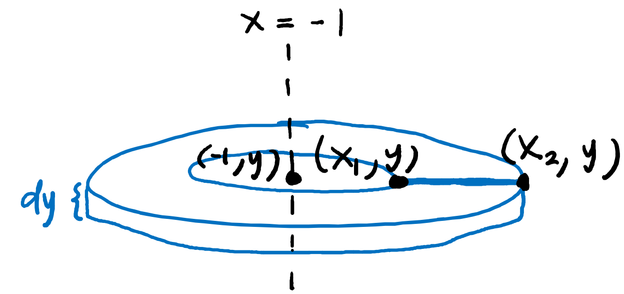
6.2 Cross-section is annulus

The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the line $x = -1$. Find the volume of the resulting solid.





cross-section is an annulus



$$R = x_2 - (-1) = x_2 + 1$$

$$r = x_1 - (-1) = x_1 + 1$$

$$\text{Area} = \pi R^2 - \pi r^2$$

$$\text{volume of slice} = (\pi R^2 - \pi r^2)\, dy$$

$$\text{Total Volume} = \int_0^1 (\pi R^2 - \pi r^2)\, dy$$

$$= \pi \int_0^1 (x_2+1)^2 - (x_1+1)^2\, dy$$

$$= \pi \int_0^1 (\sqrt{y}+1)^2 - (y+1)^2\, dy$$

$$= \pi \int_0^1 (y + 2\sqrt{y} + 1) - (y^2 + 2y + 1) \; dy$$

$$= \pi \int_0^1 2\sqrt{y} - y - y^2 \, dy$$

$$= \pi \left[\frac{4}{3} y^{3/2} - \frac{y^2}{2} - \frac{y^3}{3} \right]_0^1$$

$$= \pi \left[\left(\frac{4}{3} - \frac{1}{2} - \frac{1}{3} \right) - (0 - 0 - 0) \right]$$

$$= \frac{\pi}{2}$$