Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

8.5 Power Series

EXAMPLE 4 Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

By the Ratio Test, three things are true:

1. For values of x where $L = 3|x| < 1$, the power series converges absolutely.
2. For values of x where $L = 3|x| > 1$, the power series diverges.
3. For values of x where $L = 3|x| = 1$, the Ratio Test is inconclusive and we need to use other methods to determine convergence at these values of x.

Observe that the inequality $3|x| < 1$ contains the information about the Radius of Convergence. **When the coefficient of x is 1**, the number on the right-hand side becomes the **Radius of Convergence**:

$$3|x| < 1 \iff |x| < \frac{1}{3}$$

Therefore the **Radius of Convergence** is $R = \dfrac{1}{3}$.

To organize the convergence results from the Ratio Test, we draw a number line to indicate the **interval of convergence**: $|x| < \frac{1}{3}$ describes the values of x that are less than $\frac{1}{3}$ distance away from 0. Hence a possible interval of convergence is $\left(-\frac{1}{3}, \frac{1}{3}\right)$ and we draw

Observe that our power series is centered at 0. On the interval of convergence above, the distance from the center 0 to the boundary of the interval is $\frac{1}{3}$. In other words, the **Radius of Convergence** is the distance from the center to the boundary on the interval of convergence, and it is equal to $\frac{1}{3}$ for this example. We excluded $|x| > 1$ from the interval of convergence because the power series diverges at those values of x.

Note that the Ratio Test gives no information about the **boundary points** when $3|x| = 1$. The two boundary points $x = -\frac{1}{3}$ and $x = \frac{1}{3}$ must be checked manually using other convergence tests.

8.5 Power Series Centered at A

Definition. A series of the form

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$$

is called a **power series centered at** a.

Ratio Test for Power Series Centered at a.

Given a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$, let

$$L(x) = \lim_{n\to\infty} \left| \frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n} \right| = |x-a| \cdot \lim_{n\to\infty} \left| \frac{c_{n+1}}{c_n} \right|.$$

1. For values of x where $L(x) < 1$, the power series is absolutely convergent.
2. For values of x where $L(x) > 1$, the power series diverges.
3. For values of x where $L(x) = 1$, the Ratio Test is inconclusive and we must use other testing methods to determine convergence.

8.5 Radius of Convergence

Theorem. For a given power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only three possibilities:

1. The series converges only when $x = a$. $(R = 0)$
2. The series converges for all x. $(R = \infty)$
3. There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.

Definition. The number R is called the **radius of convergence** of the power series.

The **interval of convergence** of a power series is the interval that consists of all values of x for which the series converges.

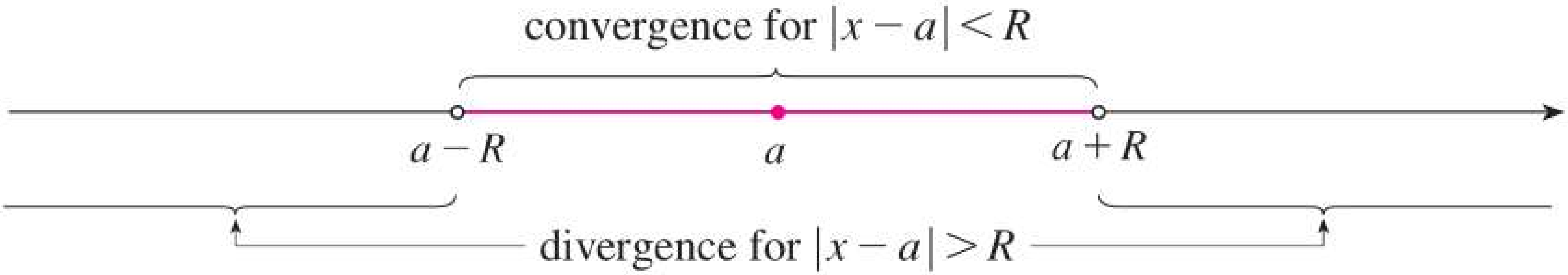
Computing Interval of Convergence

$$|x - a| < R$$

Boundary points of the Interval of Convergence

WARNING: When x is a *boundary point* of the interval, that is, $x = a \pm R$, anything can happen - the series might converge at one or both boundary points or it might diverge at both boundary points. Thus when R is positive and finite, there are four possibilities for the interval of convergence:

$$(a - R, a + R), \quad (a - R, a + R], \quad [a - R, a + R), \quad [a - R, a + R]$$



8.5 Power Series

V EXAMPLE 2 Using the Ratio Test to determine where a power series converges

For what values of x does the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ converge?

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	Series	Radius of convergence	Interval of convergence
Geometric series	$\sum_{n=0}^{\infty} x^n$	$R = 1$	$(-1, 1)$
Example 1	$\sum_{n=0}^{\infty} n!\, x^n$	$R = 0$	$\{0\}$
Example 2	$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$	$R = 1$	$[2, 4)$
Example 3	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$	$R = \infty$	$(-\infty, \infty)$

8.5 Power Series

V EXAMPLE 5 Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$$