Exam 2 Review Session

Sebastian Bozlee

Leo Herr

Jun Hong

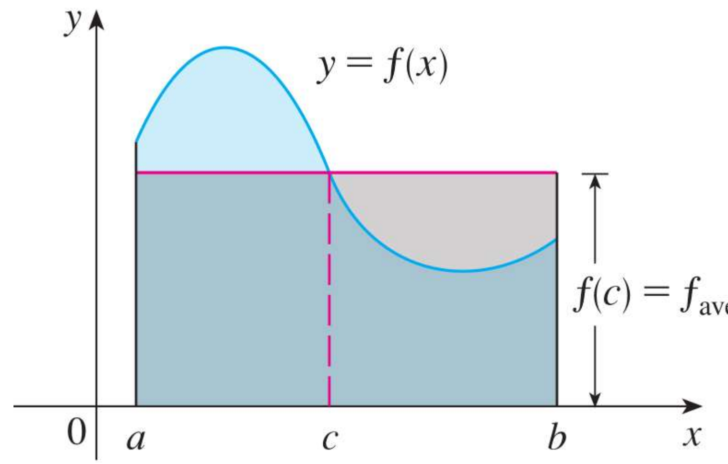
Table of Contents

Given a function $f(x)$, the **average value of** f on the interval $[a, b]$ is

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x)\ dx$$

Mean Value Theorem for Integrals. If f is continuous on $[a, b]$, then ther exists a number c in $[a, b]$ such that

$$f(c) = f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x)\, dx$$



that is,

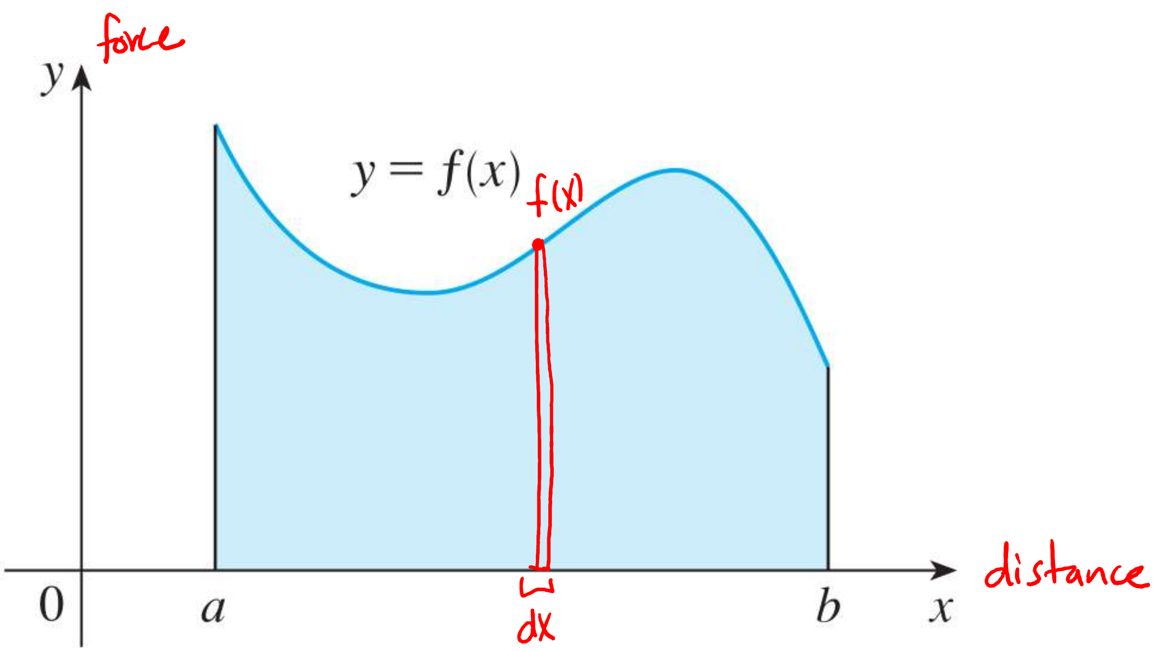
$$\int_a^b f(x)\, dx = f(c)(b-a)$$

Geometric Interpretation. For positive functions f, there is a number c such that the rectangle with base $[a, b]$ and height $f(c)$ has the same area as the region under the graph of f from a to b.

6.6 Work

kg is a unit of mass
lb is a unit of force (weight)

- Work = Force x Distance.
- If force is a function that changes with respect to distance, then work can be thought of as the area under the curve.

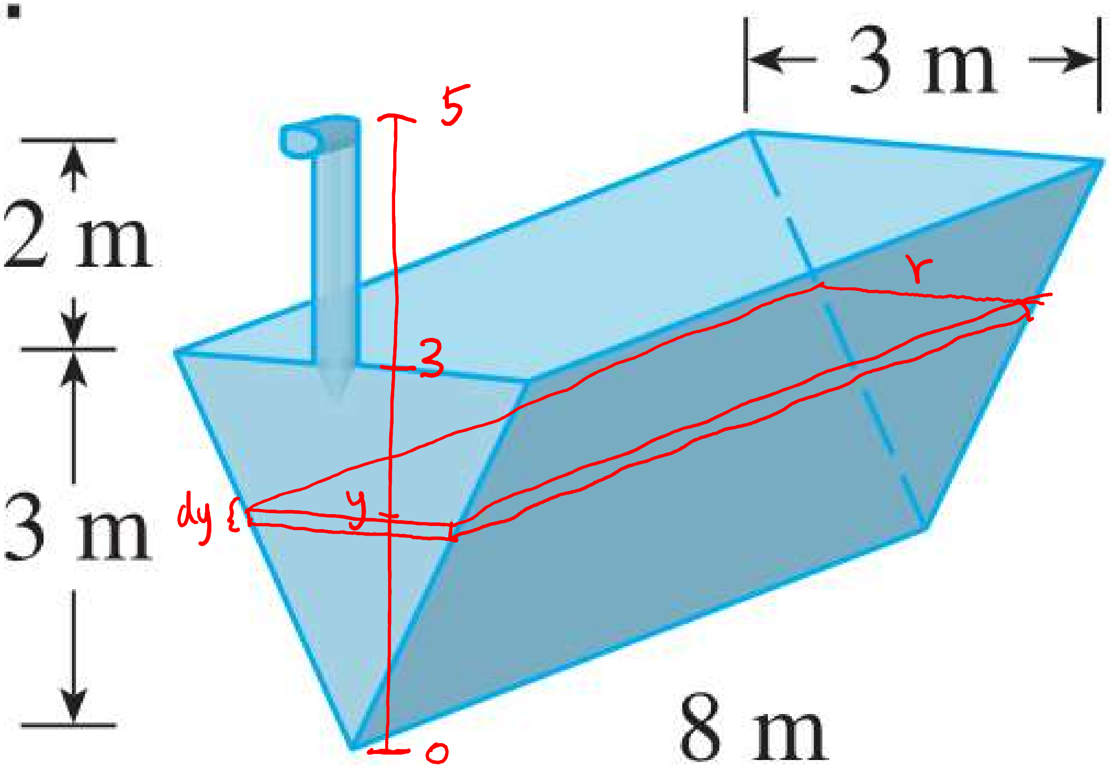


$$W = \int_a^b \underbrace{f(x)}_{\text{force}} \; \underbrace{dx}_{\text{small distance}}$$

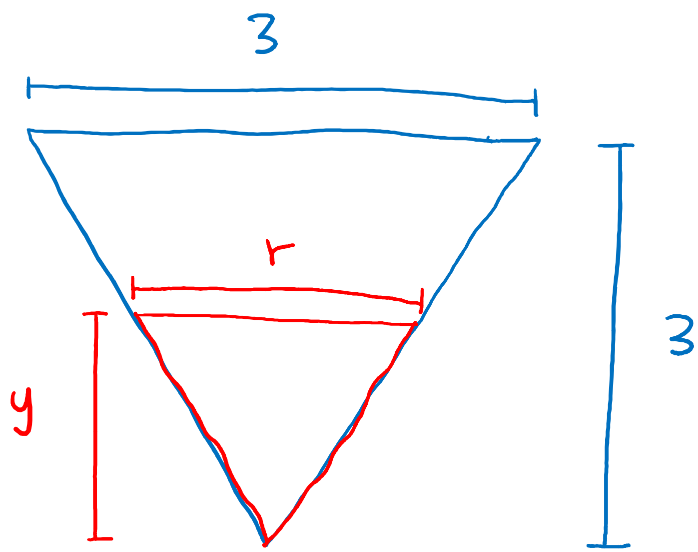
6.6 Work

A tank is full of water. Find the work required to pump the water out of the spout.

19.



Similar Triangles



$$\frac{3}{3} = \frac{r}{y}$$

$$r = y$$

volume of slice $= 8r\,dy = 8y\,dy$

weight of slice $= (8y\,dy)\,1000\,g$

distance of the slice from the spout $= 5-y$

work required to bring the slice to the spout $= 8000g\,(5-y)\,y\,dy$

Total work $= \displaystyle\int_0^3 8000g\,(5-y)\,y\,dy$

$$= 8000g \int_0^3 (5y-y^2)\,dy$$

$$= 8000g \left[\frac{5y^2}{2} - \frac{y^3}{3}\right]_0^3$$

$$= 8000g\left[\left(\frac{45}{2} - \frac{27}{3}\right) - 0\right]$$

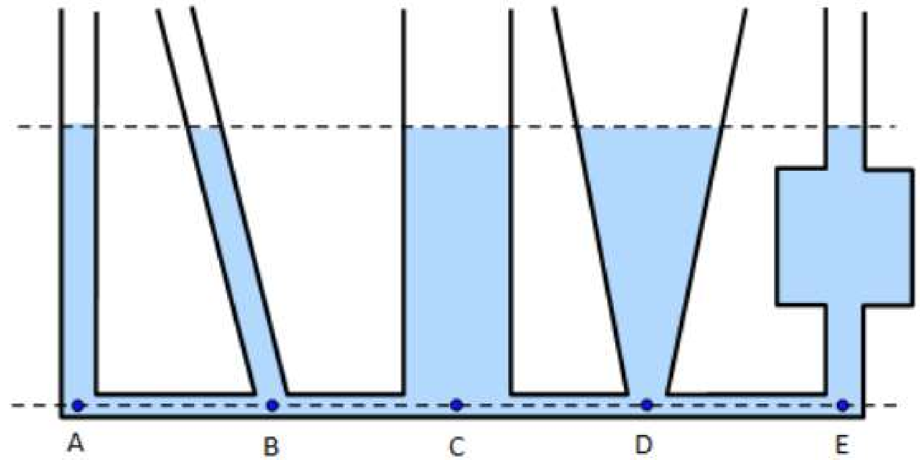
$$= 8000g\,\frac{27}{2}$$

$$= 108000\,g \text{ Joules}$$

6.6 Hydrostatic Pressure and Force

At any point in a liquid the pressure is the same in ALL directions. The pressure of a liquid is the same at any given depth below the surface regardless of the shape of the container.

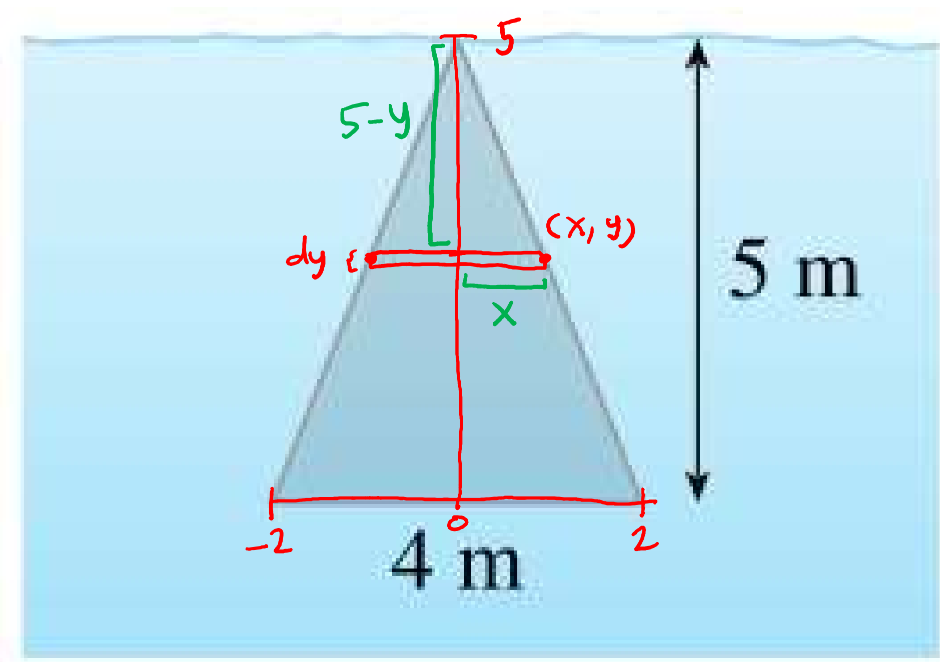
$$P = \frac{F}{A} = \rho g d$$



ρ is the density of water, g is the gravitational constant, and d is the depth of the water.

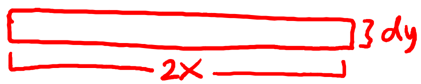
6.6 Hydrostatic Pressure and Force

A triangle with base 4 m and height 5 m is submerged vertically in water so that the tip is even with the surface. Express the hydrostatic force against one side of the plate as an integral.



$P = \rho g d$

$F = P A$



Area of slice $= 2x\,dy$

pressure on the slice $= \rho g (5-y)$

Force on the slice $= \rho g (5-y)\, 2x\, dy$

Total force

$$= \int_0^5 \rho g (5-y)\, 2x\, dy$$

Now we solve for x in terms of y.

Using similar triangles,

$$\frac{2}{5} = \frac{x}{5-y}$$

$$x = \frac{2(5-y)}{5}$$

So the total force is

$$\int_0^5 \rho g(5-y)\, 2\left(\frac{2(5-y)}{5}\right) dy \quad \text{Newton}$$

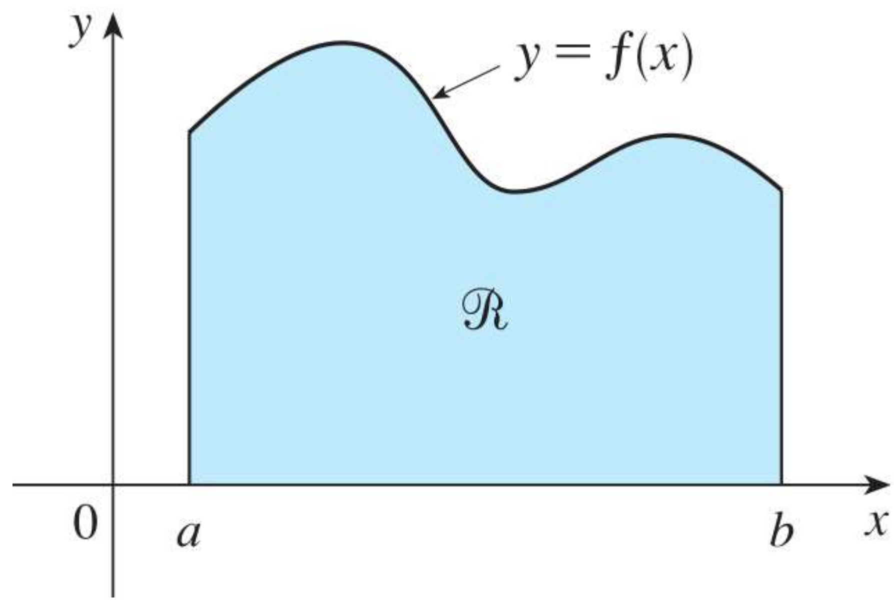
Center of Mass

The center of mass of a region $\mathscr{R}$ of constant density is located at $(\overline{x}, \overline{y})$ and

$$\overline{x} = \frac{1}{A}\int_a^b x f(x)dx$$

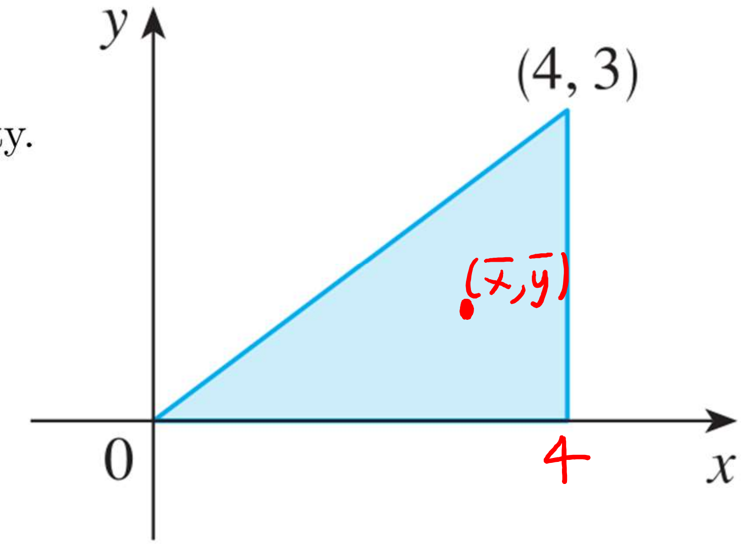
$$\overline{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2 dx$$

where $A = \int_a^b f(x)dx.$



Computing the Center of Mass

Calculate the center of mass of the given lamina with constant density.



Equation of the line : $y = \frac{3}{4}x$

$$A = \int_0^4 \frac{3}{4}x\,dx = \frac{3}{4}\left[\frac{x^2}{2}\right]_0^4 = \frac{3}{4}[8-0] = 6$$

$$\bar{x} = \frac{1}{A}\int_a^b x f(x)\,dx = \frac{1}{6}\int_0^4 x\left(\frac{3}{4}x\right)dx = \frac{1}{8}\left[\frac{x^3}{3}\right]_0^4$$

$$= \frac{1}{8}\left[\frac{64}{3} - 0\right] = \frac{8}{3}$$

$$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx = \frac{1}{6}\int_0^4 \frac{1}{2}\left(\frac{3}{4}x\right)^2 dx = \frac{1}{12}\cdot\frac{9}{16}\left[\frac{x^3}{3}\right]_0^4$$

$$= \frac{3}{64}\left[\frac{64}{3} - 0\right] = 1$$

Center of mass $(\bar{x},\bar{y}) = \left(\frac{8}{3}, 1\right)$