Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

8.4 Absolute Convergence

Given any series $\sum_{n=1}^{\infty} a_n$, we can consider the corresponding series of absolute values

$$\sum_{n=1}^{\infty} |a_n| = |a_1| + |a_2| + |a_3| + \cdots$$

whose terms are the absolute values of the terms of the original series.

Definition. A series $\sum_{n=1}^{\infty} a_n$ is called **absolutely convergent** if the series of absolute values $\sum_{n=1}^{\infty} |a_n|$ is convergent.

Example. A series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ is **absolutely convergent** if the series of absolute values $\sum_{n=1}^{\infty} b_n$ is absolutely convergent.

Remark. Notice that if $\sum_{n=1}^{\infty} a_n$ is a series with positive terms, then $|a_n| = a_n$ and so absolute convergence is the same as convergence for positive series.

8.4 Example of Absolutely Convergent Series

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \cdots$ **converges** as an alternating series by the alternating series test.

But it also **converges absolutely** because

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n-1}}{n^2} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

is a p-series with $p = 2$.

Absolute Convergence is Stronger than Convergence

Theorem. If a series $\sum_{n=1}^{\infty} a_n$ is **absolutely convergent**, then it is **convergent**.

Conditional Convergence

Definition. A series $\sum_{n=1}^{\infty} a_n$ is called **conditionally convergent** if it is **not absolutely convergent** but still **converges**.

Version for Alternating Series

Theorem. If an alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ is **absolutely convergent**, then it is **convergent**.

Conditional Convergence for Alternating Series

Definition. An alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ is called **conditionally convergent** if it is **not absolutely convergent** but still **converges**.

8.4 Conditional Convergence

Examples of conditionally convergent series:

The alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}$ converges but its series of absolute values $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges. (Why?)

The alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$ converges but its series of absolute values $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ diverges. (Why?)

8.4 Modes of Convergence

Absolutely Convergent

Definition. A series $\sum_{n=1}^{\infty} a_n$ is called **absolutely convergent** if the series of absolute values $\sum_{n=1}^{\infty} |a_n|$ is convergent.

Conditionally Convergent

Definition. A series $\sum_{n=1}^{\infty} a_n$ is called **conditionally convergent** if it is not absolutely convergent but still converges.

Divergent

Definition. A series $\sum_{n=1}^{\infty} a_n$ is divergent if the sequence of its partial sums $s_m = \sum_{n=1}^{m} a_n$ has no limit as n goes to infinity.

8.4 Absolute Convergence

V EXAMPLE 7 Determine whether the series

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^2} = \frac{\cos 1}{1^2} + \frac{\cos 2}{2^2} + \frac{\cos 3}{3^2} + \cdots$$

is convergent or divergent.

8.4 The Ratio Test

The Ratio Test

(i) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

(ii) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test is inconclusive; that is, no conclusion can be drawn about the convergence or divergence of $\sum a_n$.

8.4 The Ratio Test

Using the Ratio Test Test the series $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$ for absolute convergence.

8.4 The Ratio Test

Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$$

Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$$

Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty} \frac{n!}{100^n}$$