Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

Examples of Comparisons that You Should Know

b_n	grows slower than	a_n
$\ln(n)$		any polynomial
any polynomial of degree k		any polynomial of degree greater than k
any polynomial		any growing exponential (a^n, where $a > 1$)
b^n		a^n, where $a > b > 1$
any growing exponential		$n!$
$n!$		n^n

- Provide justification. For example,

$$\boxed{\lim_{n\to\infty} \frac{5x^3 + 2x - 4}{3^n} = 0, \text{ since exponentials grow faster than polynomials}}$$

8.4 Alternating Series

Definition. Given $b_n \geq 0$, the series

$$\sum_{n=1}^{\infty} (-1)^{n-1}\, b_n = b_1 - b_2 + b_3 - b_4 + \cdots$$

is called an **alternating series**.

8.4 Alternating Series

Example: Consider the harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots = \infty$$

Here is the **alternating harmonic series**

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$$

The original harmonic series diverges to infinity but the alternating harmonic series **converges**!

8.4 Examples of Convergent Alternating Series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{1}{1} - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \cdots$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} = \frac{1}{1} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \cdots$$

8.4 Graphical Demonstration of Convergence of the Alternating Harmonic Series

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$$

Desmos

8.4 Alternating Series Test

Alternating Series Test. Suppose $\sum_{n=1}^{\infty}(-1)^{n-1}\, b_n$ is an alternating series.

If b_n satisfy the two conditions

(a) $\lim_{n\to\infty} b_n = 0$ (vanishing at infinity)

(b) $b_n \geq b_{n+1}$ (decreasing)

then the alternating series is convergent.

8.4 Alternating Series Test

Show that the alternating harmonic series converges.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$$

Let's use the alternating series test. We need to check the two hypotheses.

① Vanishing at infinity

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n}$$

$$= 0$$

② Decreasing

$b_n = \frac{1}{n}$, $f(x) = \frac{1}{x}$, $f'(x) = -\frac{1}{x^2}$

Since $f'(x) = -\frac{1}{x^2} < 0$ for $x \geq 1$,

$b_n = \frac{1}{n}$ is a decreasing sequence.

The series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ satisfies the hypotheses of the alternating series test so it is convergent.

8.4 Alternating Series

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$$

Use the alternating series test. Check the hypotheses

① Vanishing at infinity

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}}$$

$$= 0$$

② Decreasing

$b_n = \frac{1}{\sqrt{n}}$, $f(x) = \frac{1}{\sqrt{x}}$, $f'(x) = \frac{-1}{2x^{3/2}}$

Since $f'(x) = \frac{-1}{2x^{3/2}} < 0$ for $x \geq 1$,

$b_n = \frac{1}{\sqrt{n}}$ is a decreasing sequence.

The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ satisfies the hypotheses of the alternating series test so it is convergent.

8.4 Alternating Series

Test the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{3n}{4n-1}$ for convergence or divergence.

Try the AST.

① Vanishing at infinity

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{3n}{4n-1}$$

$$= \frac{3}{4} \neq 0$$

Since the sequence b_n doesn't vanish at infinity, the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{3n}{4n-1}$ diverges by the Divergence Test.

8.4 Alternating Series

Test the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{n^3+1}$ for convergence or divergence.

Try the AST. Check the hypotheses.

① Vanishing at infinity

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n^2}{n^3+1}$$

$$= 0$$

② Decreasing

$$b_n = \frac{n^2}{n^3+1}, \quad f(x) = \frac{x^2}{x^3+1},$$

$$f'(x) = \frac{2x(x^3+1) - x^2(3x^2)}{(x^3+1)^2}$$

$$f'(x) = \frac{2x - x^4}{(x^3+1)^2}.$$

Since $f'(x) = \dfrac{2x-x^4}{(x^3+1)^2} < 0$ for $x > \sqrt[3]{2}$, $b_n = \dfrac{n^2}{n^3+1}$ is a decreasing sequence.

The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$ satisfies the hypothesis of the alternating series test so it is convergent.