Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

8.3 Limit Comparison Test

The Limit Comparison Test. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

If $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists and is non-zero, then either both series converge or both series diverge.

8.3 Limit Comparison Test

EXAMPLE 5 Using the Limit Comparison Test

Test the series $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ for convergence or divergence.

$\frac{1}{2^n-1}$ looks similar to $\frac{1}{2^n}$. Let's use the Limit Comparison Test.

First, check the hypothesis. Since both $\frac{1}{2^n-1}$ and $\frac{1}{2^n}$ are positive for $n \geq 1$, the hypothesis for LCT is satisfied. Taking the limit,

$$\lim_{n\to\infty} \frac{\frac{1}{2^n-1}}{\frac{1}{2^n}} = \lim_{n\to\infty} \frac{2^n}{2^n-1} = \lim_{n\to\infty} \frac{1}{1-\frac{1}{2^n}} = \frac{1}{1-0} = 1$$

Since the limit of the ratio is a number greater than 0, by the LCT either both $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$ and $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converge or both diverge.

But $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges since it is geometric with $r = \frac{1}{2}$, $|r| = \frac{1}{2} < 1$.

Hence $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ also converges.

8.3 Limit Comparison Test

Determine whether the series $\sum_{n=1}^{\infty} \frac{5}{2n^2 + 4n + 3}$ converges or diverges.

$\frac{5}{2n^2+4n+3} \approx \frac{1}{n^2}$ and since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-test $(p=2)$, we guess that $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ converges. Let's use the Limit Comparison Test.

First check the hypothesis of the LCT.

Hypothesis: Both $\frac{5}{2n^2+4n+3}$ and $\frac{1}{n^2}$ are positive for $n \geq 1$.

Then we can proceed and take the limit.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{5}{2n^2+4n+3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{5n^2}{2n^2+4n+3} = \frac{5}{2}$$

Since the limit of the ratio is a number that is greater than 0, the Limit Comparison Test says that either both series converge or both diverge. Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-test $(p=2)$, $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ also converges.

Examples of Comparisons that You Should Know

b_n	grows slower than	a_n
$\ln(n)$		any polynomial
any polynomial of degree k		any polynomial of degree greater than k
any polynomial		any growing exponential (a^n, where $a > 1$)
b^n		a^n, where $a > b > 1$
any growing exponential		$n!$
$n!$		n^n

- Provide justification. For example,

$$\boxed{\lim_{n\to\infty} \frac{5x^3 + 2x - 4}{3^n} = 0, \text{ since exponentials grow faster than polynomials}}$$