1. Determine the degree of the Taylor polynomial $P_n(x)$ expanded about $c = 1$ that should be used to approximate $\ln(1.2)$ so that the error is less than 0.0005.

   (a) Find the first four derivatives for $f(z) = \ln(z)$

   (b) Find a formula for $\left|f^{(n+1)}(z)\right|$ where $z > 0$.

   (c) We are approximating the function $f(x) = \ln(x)$ at $x = 1.2$ using a Taylor polynomial centered at $c = 1$. If

   $$|f^{(n+1)}(z)| \leq M$$

   for all $z$ in the interval $1 \leq z \leq 1.2$. Then we wish to find the upper bound $M$. To do this find

   $$\frac{d}{dz}\left(|f^{(n+1)}(z)|\right).$$

   i. Does $|f^{(n+1)}(z)|$ have any critical points on $1 \leq z \leq 1.2$?

   ii. Is $|f^{(n+1)}(z)|$ increasing or decreasing on $1 \leq z \leq 1.2$?

(d) Use part (c) to identify a good upper bound $M$ for $|f^{(n+1)}(z)|$ on the interval $1 \leq z \leq 1.2$. (Notice that $M$ will depend on $n$.)

(e) Make use of the inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-c|^{n+1} < 0.0005$$

to find an appropriate value for $n$ that gives accuracy within 0.0005.

(f) Finally approximate $\ln(1.2)$ using the $n$th-degree Taylor polynomial centered at $c = 1$ with the $n$ you found in part (e).