Even when we stay within the realm of elementary functions, the general families of integrals that come up can be quite varied. So when integrating, it will often be practical to use a table of integrals or a computer algebra system (Wolfram Alpha, Maple, Mathematica, etc.), rather than work out an integral by hand. However, antiderivatives are only unique up to a constant, so different methods may produce answers that look quite different.

1. In this question we will investigate the integral $\displaystyle\int \frac{dx}{\sqrt{9+x^2}}$.

   (a) Compute the antiderivative using a trig substitution.

   (b) What is the antiderivative according to the table of integrals in your book? Can you make the two answers agree?

   (c) What does Wolfram Alpha say this integral is? Is Wolfram wrong?

2. The function $\sinh x$, pronounced "cinch x", is the hyperbolic sine function. So $\sinh^{-1} x$ (also known as arcsinh$x$) is the inverse of $\sinh x$.

   (a) Use the internet to determine the definition of $\sinh x$ in terms of exponential functions.

   (b) Use the internet to determine the logarithmic representation of $\sinh^{-1} x$.

   (c) Verify that your answers to 1(a) and 1(b) agree with Wolfram Alpha's answer.

3. Compute $\displaystyle\int \frac{1}{x^2-1}dx$:

   (a) Using partial fractions

(b) Using the tables in your book. Then show that the two answers actually agree.

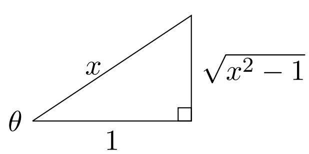
(c) We can also approach this problem with a trig substitution. We see the form $x^2 - a^2$, and looking at the pythagorean identity $\tan^2\theta = \sec^2\theta - 1$ suggests that we could use a secant substitution.

$$\int \frac{dx}{x^2-1} \quad \begin{cases} x = \sec\theta \\ dx = \sec\theta\tan\theta d\theta \end{cases} \quad \Longrightarrow \quad \int \frac{\sec\theta\tan\theta d\theta}{\sec^2\theta - 1}$$

$$= \int \frac{\sec\theta\tan\theta d\theta}{\tan^2\theta} = \int \frac{\sec\theta d\theta}{\tan\theta}$$

$$= \int \csc\theta d\theta = -\ln|\csc\theta + \cot\theta| + C$$

Now complete the integral by using the triangle below to substitute back $\theta =$ ______________



(d) Use algebraic manipulation to show that this third answer agrees with the answers from parts (a) and (b).

4. Find the number of the form in the table of integrals in your book that you can use to find each of the following integrals, and give the correct antiderivative.

   (a) $\displaystyle\int \frac{x^2+25}{\sqrt{x^2+25}}dx$

   (b) Use the table of integrals to inspire a choice for a u/du substitution. Make the substitution, then complete the integral using the table. $\displaystyle\int \frac{\csc^2(3x)dx}{\sqrt{12+\tan^2(3x)}}$

We can see that using tables of integrals, as well as interpreting the results of a computer algebra system integration requires the ability to recognize and rewrite functions in various forms. If you find yourself using these methods to compute an integral, just remember that with a little work and perhaps some research, you can make sense of an unfamiliar answer!