Background: We've calculated the Taylor Series centered about $a = 0$ for some important functions. For each of these functions, we know that if it has a power series representation, then it must be actually equal to its Taylor Series on the interval of convergence. In fact, each of the functions below is equal to its Taylor Series, facts that we will show next week. It's worth remembering the Taylor Series for the functions below, in both expanded notation and sigma notation.

$e^x =$ ____________________ $=$ __________ on the interval __________

$\sin x =$ ____________________ $=$ __________ on the interval __________

$\cos x =$ ____________________ $=$ __________ on the interval __________

$\dfrac{1}{1-x} =$ ____________________ $=$ __________ on the interval __________

We will now build new Taylor series from familiar old ones using the techniques of substitution, multipication, differentiation and integration.

1. (a) Write down the Taylor series centered about $a = 0$ for $f(x) = e^x$ in expanded form. Take the derivative. What do you notice? Why does this make sense for $f(x)$?

   (b) Write down the Taylor series centered about $a = 0$ for $f(x) = e^x$ in summation notation (Σ-notation). Take the derivative. Confirm that you get the Taylor series for e^x again.

2. (a) Write down the Taylor series centered about $a = 0$ for $f(x) = \sin x$ in expanded form. Take the derivative. What do you notice? Why does this make sense for $f(x)$?

   (b) Write down the Taylor series centered about $a = 0$ for $f(x) = \cos x$ in summation notation (Σ-notation). Take the derivative. Confirm that you get the Taylor series for the derivative of $\cos x$.

3. Find the Taylor series centered about $a = 0$ for $f(x) = \cos 2x$. Derive the answer working in expanded notation and then again in Σ-notation. Confirm the two answers are the same.

4. Find the Taylor Series centered about $a = 0$ for $x^2 e^{3x}$. Give the answer in Σ-notation.

5. Find the Taylor Series centered about $a = 0$ for $\sin x^2$.

6. Find the Taylor Series centered about $a = 0$ for $\dfrac{e^x - 1}{x}$.

7. Find the Taylor series centered about $a = 0$ for $\dfrac{1}{1-x}$

8. Find the Taylor series centered about $a = 0$ for $\dfrac{1}{(1-x)^2}$

9. Find the Taylor series centered about $a = 0$ for $\dfrac{1}{1+t^2}$

10. Find the Taylor series centered about $a = 0$ for $\arctan x = \displaystyle\int_0^x \frac{1}{1+t^2}\,dt$