For each of these integrals, determine a strategy for evaluating. Don't evaluate them, just figure out which technique of integration will work, including what substitutions you will use.

1. $\int x\sqrt{9-x^2}\,dx$

2. $\int x^2\sqrt{9-x^2}\,dx$

3. $\int \sin^6 x\cos^2 x\,dx$

4. $\int \sin^5 x\cos^2 x\,dx$

5. $\int \frac{3}{x^2+5x+4}\,dx$

6. $\int \frac{3}{x^2+6x+9}\,dx$

7. $\int \arcsin x\,dx$

8. $\int \frac{\arctan x}{1+x^2}\,dx$

9. $\int x\sqrt{x+2}\,dx$

10. $\int (x+2)\sqrt{x}\,dx$

11. $\int \frac{e^x}{4+e^{2x}}\,dx$

12. $\int \frac{1}{x\ln x}\,dx$

13. $\int x^2\cos 5x\,dx$

14. $\int \frac{x^2+1}{x}\,dx$

15. $\int \frac{x+5}{x^2+4}\,dx$

16. $\int \tan^4 x\sec^2 x\,dx$

Answers:

1. u/du substitution, with $u = 9 - x^2$, $du = -2x\,dx$
2. Trig substitution, $x = 3\sin\theta$
3. Both powers are even, use the power reduction formulas ($\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$)
4. The power on $\sin x$ is odd, so let $u = \cos x$, $du = -\sin x\,dx$, convert remaining powers of $\sin x$ using $\sin^2 x = 1 - \cos^2 x$
5. Partial fractions, $= \frac{A}{x+4} + \frac{B}{x+1}$
6. Factoring gives $\frac{3}{(x+3)^2}$, let $u = x + 3$, $du = 3\,dx$
7. Integration by parts, $u = \arcsin x$, $dv = dx$. The integral we're left with is $\int \frac{x}{\sqrt{1-x^2}}\,dx$, use $u = 1 - x^2$ and $du = -2x\,dx$ on that.
8. $u = \arctan x$, $du = \frac{1}{1+x^2}\,dx$
9. $u = x + 2$, $du = dx$. Then distribute and use the power rule
10. Just distribute, then the power rule works
11. $u = e^x$, $du = e^x\,dx$. What results is an arctan function.
12. $u = \ln x$, $du = \frac{1}{x}\,dx$
13. Integration by parts, $u = x^2$, $dv = \cos 5x\,dx$. Integration by parts will be repeated on the resulting integral.
14. Just simplify algebraically.
15. Break it into two integrals, $\int \frac{x}{x^2+4}\,dx + \int \frac{5}{x^2+4}\,dx$. Use $u = x^2 + 4$ on the first integral, and the second integral is just an arctan function.
16. $u = \tan x$, $du = \sec^2 x\,dx$