SAMPLE MIDTERM I
EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY

MATH 3210

Friday February 14, 2014

Name | ______________________________

Please answer all of the questions, and show your work.
All solutions must be explained clearly to receive credit.

Date: February 10, 2014.

1
10 points

1. Suppose A, B, C are subsets of a set X. For each of the following statements, either prove the statement, or provide a counter example.

1.(a). $A \cup (B \cap A^c) = A \cup B$.

1.(b). $A - (B \cup C) = (A - B) \cup (A - C)$.

1.(c). $(A \cap B) \cup (A \cap B^c) = A$.

2
10 points

2. Let $X = \mathbb{Z} \times (\mathbb{Z} - \{0\})$. Define a relation on X by

$$(a, b) \sim (c, d) \iff ad = bc.$$

2.(a). Show that $\sim$ is an equivalence relation.

2.(b). Show that if $(a, b) \sim (a', b')$ and $(c, d) \sim (c', d')$, then $(ac, bd) \sim (a'c', b'd')$.

2.(c). Show that if $(a, b) \sim (a', b')$ and $(c, d) \sim (c', d')$, then $(ad + bc, bd) \sim (a'd' + b'c', b'd')$.

2.(d). Let $Q = X/\sim$ be the quotient of X by the equivalence relation. Show that there is a map

$$\mu : Q \times Q \to Q$$

given by $\mu([(a, b)], [(c, d)]) = [(ac, bd)]$.

2.(e). Show that there is a map

$$\mu : Q \times Q \to Q$$

given by $\alpha([(a, b)], [(c, d)]) = [(ad + bc, bd)]$.

3
10 points

3. Let S be a non-empty set and let $\mathrm{Map}(S, S)$ be the set of maps from S to S. Suppose that for every $f, g \in \mathrm{Map}(S, S)$, we have

$$f \circ g = g \circ f.$$

Show that S has only one element.

4
10 points

4. Give the definition of a Classical Euclidean Plane.

5
10 points

5. Let $\mathbb{P}$ be a Classical Euclidean Plane. Show that if L, M are distinct non-parallel lines, then L and M have a unique point in common.

6
10 points

6. Given an incidence plane, show that there exist three distinct, non-concurrent lines.

7
10 points

7. For each $(a, b, c) \in \mathbb{R}^3$, define

$$L(a, b, c) := \{(x, y) \in \mathbb{R}^2 : ax + by + c = 0\}.$$

Define

$$\mathcal{P} = \mathbb{R}^2,$$
$$\mathcal{L} = \{L(a, b, c) : [(a, b, c) \in \mathbb{R}^3] \wedge [(a \neq 0) \vee (b \neq 0)]\}$$

and

$$\lambda : \mathcal{P} \times \mathcal{L} \to \{0, 1\}$$
$$\lambda(p, \ell) = 1 \iff p \in \ell.$$

Show that $(\mathcal{P}, \mathcal{L}, \lambda)$ is an incidence plane.