SAMPLE
MIDTERM II
INTRODUCTION TO LINEAR ALGEBRA

MATH 3130

Friday March 22, 2013

Name | ______________________________

Please answer all of the questions, and show your work.
All solutions must be explained clearly to receive credit.

1	2	3	4	5	6	7	8	
10	10	20	20	10	10	10	10	Total

Date: March 17, 2013.

1
10 points

1.(a). [3 points] What is the definition of a symmetric matrix?

1.(b). [3 points] What is the definition of an invertible matrix?

1.(c). [4 points] Let A be a symmetric, invertible, $n \times n$ matrix. Use the definition of a symmetric matrix and the definition of an invertible matrix to prove that A^{-1} is also symmetric.

2
10 points

2. Find the determinant of each of the following matrices.

2.(a). $A = \begin{bmatrix} 4 & -1 & 1 \\ -1 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

2.(b). $B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \pi \\ 1 & 0 & e & -4 & 8 & 3^{-5} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 & 2 & 10^4 \\ 0 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & -1 & 2 & 0 \end{bmatrix}$

3
20 points

3. Consider the matrix $C = \begin{bmatrix} 2 & 4 & -3 \\ 1 & -1 & 2 \\ 2 & 3 & -1 \end{bmatrix}$.

3.(a). Find the cofactor matrix of C.

3.(b). Find the inverse of C using the cofactor matrix from part (a).

4
20 points

4. Put the matrix $D = \begin{bmatrix} -1 & -4 & -2 & -2 & -14 \\ 3 & 12 & 1 & 3 & 22 \\ 2 & 8 & 0 & 2 & 14 \end{bmatrix}$ into reduced row echelon form.

5
10 points

5. Let D be the matrix in the previous problem.

5.(a). [3 points] Find a basis for the row space of D.

5.(b). [3 points] Find a basis for the column space of D.

5.(c). [4 points] Find a basis for the kernel of D.

6
10 points

6. The equation $x = Cx + d$ (the Leontief Production Equation) arises in the Leontief Input-Output Model. Here $x, d \in M_{n\times 1}(\mathbb{R})$ and $C \in M_{n\times n}(\mathbb{R})$. Consider also the equation $p = C^T p + v$ (called the price equation), where $p, v \in M_{n\times 1}(\mathbb{R})$. *Show that*

$$p^T d = v^T x.$$

(This quantity is known as GDP.) [Hint: Compute $p^T x$ in two ways.]

7
10 points

7. Let $V = \mathbb{R}[x]$ be the vector space of real polynomial functions. Let $D : V \to V$ be the derivative map; i.e. $D(p) = p'$ for all $p \in V$. Let $E : V \to V$ be the integration map that sends a polynomial p to the polynomial q given by $q(x) = \int_0^x p(t)dt$, for all $x \in \mathbb{R}$. It is a fact that D and E are linear maps.

7.(a). Show that D is surjective, but not injective.

7.(b). Show that E is injective, but not surjective.

8
10 points

8. True or False. Mark **T** for true and **F** for false.

8.(a) Let $f : V \to V$ be a linear map of a vector space to itself. If f is surjective, then f is an isomorphism.

8.(b) A square matrix A is called an orthogonal matrix if $AA^T = \mathrm{Id}$. If A and B are orthogonal, then AB is orthogonal.

8.(c) If two rows of a square matrix A are equal, then $\det(A) = 0$.

8.(d) If A and B are $n \times n$ matrices, then $\det(AB) = \det(A)\det(B)$.

8.(e) If the determinant of a square matrix is zero, then the determinant of its cofactor matrix is zero.

8.(f) If A is an $n \times n$ matrix, then the equation $Ax = b$ has at least one solution for every $b \in \mathbb{R}^n$.

8.(g) If the columns of a square matrix A are linearly independent, then A^T is invertible.

8.(h) Let A be an $m \times n$ matrix. Then there exists an $m \times m$ permutation matrix P, an $m \times m$ lower triangular matrix L and an $m \times n$ upper echelon form matrix U, such that $PA = LU$.

8.(i) If A is a square matrix such that the column sums of the absolute values in A are less than 1, then $(\mathrm{Id} - A)$ is invertible.

8.(j) An $n \times n$ matrix is invertible if and only if its column rank is strictly less than n.