Homework 7

Due Friday, December 1

Exercises

1. For a smooth curve C of genus $g \geq 4$, show directly (without using the Kempf, Kleiman-Laksov existence theorem) that W^1_{g-1} is non-empty.

2. Let C be a smooth plane quintic, with $\mathcal{O}_C(1) = \mathcal{O}_{\mathbb{P}^2}(1)|_C$ the hyperplane bundle. Show that set theoretically

$$\begin{aligned} W^2_5 &= \{\mathcal{O}_C(1)\}, \\ W^1_5 &= \{\mathcal{O}_C(1) \otimes \mathcal{O}_C(p-q) : p.q \in C\} \\ &= V_1 + \mathcal{O}_C(1), \end{aligned}$$

   where V_1 is the image of the difference map $\phi_1 : C \times C \to Pic^0(C)$.

3. With the same notation as the previous problem, let (JC, Θ) be the Jacobian of C. Show that for a suitable translate of Θ,

$$\begin{aligned} \Theta_{sing} &= V_1 + \mathcal{O}_C(1), \\ (\Theta_{sing})_{sing} &= \{\mathcal{O}_C(1)\}, \\ \mathbb{P}C_{\mathcal{O}_C(1)}(\Theta_{sing})_{sing} &= \phi_K(C), \end{aligned}$$

   the canonical model of C.