Homework 3

Due Friday, October 20

Exercises

1. Let Λ be a lattice of maximal rank in a complex vector space V of dimension g. Let H be a positive definite Hermitian form on V, such that $ImH(\Lambda, \Lambda) \subseteq \mathbb{Z}$, let $E = ImH|_\Lambda$ be the induced alternating form on Λ, and let $\lambda_1, \ldots, \lambda_{2g}$ be a symplectic basis for Λ with respect to E. Show $\lambda_1, \ldots, \lambda_g$ generate V as a complex vector space.

2. Let $c_1(L)$ be a polarization of an abelian variety X. Show that there is an isogeny $f : X \to Y$ and a principal polarization $c_1(M)$ on Y such that $f^*c_1(M) = c_1(L)$.

3. Let L be a line bundle on a complex torus X. Let

$$K(L)_0 = [\ker(\phi_L : X \to \hat{X})]_0,$$

be the connected component of the identity of the kernel. Let

$$p : X \to \overline{X} := X/K(L)_0$$

be the quotient map. Show that there is a line bundle $\overline{L}$ on $\overline{X}$ such that $p^*\overline{L} \cong L$ if and only if $L|_{K(L)_0}$ is trivial.

4. Show that if $\overline{L}$ exists, then it is nondegenerate, and $h^0(L) = h^0(\overline{L})$.

5. Show that if there exists a nonzero divisor $D \in |L|$, then $\overline{L}$ exists and is an ample line bundle on $\overline{X}$.

6. Let X be a simple abelian variety; i.e. X admits no nontrivial abelian subvariety. Show that any algebraic subvarieties V and W with

$$\dim V + \dim W \geq \dim X$$

have a nonempty intersection.