Math 3510, HW #11

1. Let $\bar{\mu}$ be the sample mean from a data set of size n for which the mean μ is unknown, while for magical reasons the variance is known to be $\sigma^2 = 10$. Determine n so that the probability that the random interval $(\bar{\mu} - \frac{1}{2}, \bar{\mu} + \frac{1}{2})$ contains μ is approximately .95.

2. Let X be a $Bin(300, p)$ random variable. You observe a value of 75 for X. Find a 90% confidence interval for p.

3. Imagine you are observing a Binomial random variable Y with parameters n and p. You want to test the hypothesis $H_0 : p = 1/2$ versus the alternative that $H_1 : p > 1/2$. You decide you'll reject H_0 if $Y > c$ for some"big" number c. Find c and n so that the probability you reject H_0 when it is actually true (when $p = 1/2$) is 0.1 and the probability you reject H_0 when it is false and the true value of $p = 2/3$ is 0.95.

4. You have the following random sample of size $n = 15$ which you wish to model as a $N(\mu, \sigma^2)$ population:

$$2.0, -0.5, 1.4, -2.2, 0.3, -0.8, 3.7, -0.1, 0.6, 0.2, 0.9, -0.1, -2.2, -1.5, 2.1.$$

(a) Give estimators of μ and σ.
(b) Find a 95% confident interval for μ.

5. Show that the sample mean from a Poisson population of parameter λ of size n is a maximum likelihood estimator for λ. Is it unbiased and consistent?

6. Consider the density function

$$f(x; \theta_1, \theta_2) = \frac{1}{\theta_1} e^{(x-\theta_2)/\theta_1} \text{ for } x < \theta_2,$$

with $f(x; \theta_1, \theta_2) = 0$ for $x \geq \theta_2$. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from this distribution. Find the method of moments estimators for θ_1 and θ_2.