RMT and Wireless Communications

Titles and abstracts

"Notes on finite-range dependent random matrices"
Greg Anderson (U. Minnesota)

We will discuss a couple of papers (joint with O. Zeitouni, imminently to appear in CPAM and Annals of Statistics) concerning LLN's and CLT's for random matrices with dependent entries. In these papers the main tools are combinatorial. We will also discuss some more recent work of the speaker (in progress) attacking similar models with quite different Stieltjes-based methods. We will compare and contrast this work with a number of other recent works in the area, sketch in some history, try to reconcile some not-obviously-compatible viewpoints, and finish up with some questions.

"Spectra of matrices with large blocks"
Wlodek Bryc (U. Cincinnati)

Block matrices arise in a frequency selective slow-fading channel in a MIMO system. Spectra of such block matrices converge almost surely, if the size of the blocks tends to infinity. I will present a method for calculating the limit of the eigenvalue distribution from a system of equations which is solved numerically. The system of equations arises naturally as an application of operator-valued free probability, but can also be derived by "elementary methods" or from formulas in Girko's work.

Based on a joint paper with Reza Rashidi-Far, Tamer Oraby, and Roland Speicher (IEE IT 2008)

"Information, estimation and thermodynamics in communication systems"
Dongming Guo (Northwestern)

"Making Free Deconvolution Practically Useful"
Merouane Debbah (Supelec)

In this talk, we review some important results on free deconvolution and their application to wireless communications. In many situations, engineers are faced with the problem of extracting information from the network. As it will be shown, this corresponds in many respect to infer on the spectrum of functionals of random matrices with only a limited knowledge on the statistics of the matrix entries. In its full generality, the problem requires some sophisticated tools related to free probability and the explicit spectrum (complete information) can hardly be obtained (except for some trivial cases). Unfortunately, the advanced theoretical framework had led the community to the misconception that the tool has no practical application. This talk takes the opposite view and shows how the free cumulants approach provides the right shift from theory to practice.

"On some asymptotic questions in RMT related to the determinants of certain operators"
Torsten Ehrhardt (UC Santa Cruz)

The gap probablities for the level spacing distributions of the eigenvalues of random matrices can be expressed as the determinants of certain integral operators. The concrete integral operators depends on the underlying random matrix ensemble and the scaling regime. Essentially the same is true for the probability densitity function of linear statistics of random matrices.

The question about the asymptotics of the gap probabilities or linear statistics thus leadx to the question about the asymptotics of determinants. In some cases, the asymptotics are known, in others they were recently computed, and in still other cases the problem is open.

In my talk I will explain this relationship and indicate how the asymptotics can be computed in some cases.

"Rate-diversity trade-off for the MIMO half-duplex relay"
Olivier Lévêque

Random matrix theory has proven recently to be a powerful tool for studying the performance of multiple-input multiple-output (MIMO) communication systems. Let us for example consider the situation where two users, each equipped with multiple antenna devices, wish to communicate over a fading non-ergodic channel. It has been shown in this case that the knowledge of the joint eigenvalue distribution of the channel matrix allows to characterize the optimal trade-off between the chosen communication rate and the rate of decrease (or ”diversity”) of the outage probability at high SNR.

In this talk, I will briefly review the above mentioned result and also consider the slightly more involved situation where the two users are helped by a third one (the ”relay”, also equipped with multiple antennas) for establishing communication. In order to characterize of the optimal rate-diversity trade-off in this case, new results from random matrix theory are required. The optimal trade-off depends on whether the relay is able to transmit and receive at the same time (full-duplex scenario) or not (half-duplex scenario).

“Jacobi matrices and wireless communication”

Nathan Levy (Technion)

The Wyner model has been introduced in 1994 in order to study the consequences of multi cell processing and provide analytical results. We study the uplink of Wyner-type models under a general fading. This model is characterized by short-range inter-cell interference, and therefore its study requires the understanding of the spectrum large random Jacobi matrices, that is matrices whose elements are zero except in the neighbourhood of the diagonal.

Toward this end, we adapt tools coming from the theory of random Schrödinger operators; we derive a new version of the Thouless formula for the strip that allows us to relate the per-cell sum-rate capacity of the communication channel to the top Lyapunov exponent of a product of appropriate random matrices. This allows us to derive convergence results and to give CLT and Large Deviations results.

“Probabilistic Analysis of Semidefinite Relaxation for Binary Quadratic Minimization with Application to Multiuser Detection”

Zhi-Quan (Tom) Luo (U. Minnesota)

We consider semidefinite programming relaxation (SDR) of a binary quadratic minimization problem. This NP-hard problem arises naturally in the maximum-likelihood detection of discrete signals for digital communications. We analyze the average performance of SDR algorithms for a class of randomly generated binary quadratic minimization problems. Although the SDR worst-case approximation ratio is unbounded for this NP-hard problem, our analysis shows that SDR can provide in polynomial time a provably near-optimal solution, achieving a constant factor approximation of the optimal objective value in probability. Moreover, this constant factor remains bounded with increasing problem size. Our proof is based on an asymptotic analysis of Karush-Kuhn-Tucker optimality conditions using random matrix theory. This is a joint work with Mikalai Kisialiou

"Two and Three Correlation Functions for Eigenvalues of Random Matrices"
James A. Mingo (Queen's University)

We consider the two and three point correlation functions of some standard ensembles of random matrices. While for finite N one can obtain explicit expressions, the expressions are often quite complicated. However in the large N limit considerable simplifications are possible, as for example with the semi-circle law for the one point function. We shall present some examples where explicit examples are possible. This is joint work with Roland Speicher.

"The uses of sparse random matrices"
Andrea Montanari (Stanford)

Standard random matrix theory deals with matrices with a positive fraction of non vanishing entries. I will suvey models for sparse random matrices, the mathematical tools to analyze them and their potential uses in communications and statistical learning.

"Minimization of quadratic forms in wireless communications"
Ralf Müller (NTNU)

The problem of minimizing $\langle x|J|x\rangle$ over a certain set of vectors $\langle x|$ is considered where J is a random matrix with given R-transform. This problem has applications in transmitter processing of wireless communications. Generalizations of that problem that are still open, but also of interest are discussed as well.

“Invertibility of random matrices”
Mark Rudelson (U. Missouri)

The invertibility of a matrix can be quantitatively characterized by its distance to the set of the singular matrices, i.e. by its smallest singular value. Consider a random matrix with independent entries satisfying a certain tail decay condition. We show that the tail distribution of the smallest singular value obeys a universal lower bound, and this bound is exact.

The proof uses a delicate estimate, showing that the probability that the weighted sum of i.i.d random variables is small is determined by the arithmetic structure of the sequence of weights. This phenomenon will be discussed as well.

This is a joint work with Roman Vershynin.

“Applications and fundamental results on random Vandermonde matrices”
Oyvind Ryan (U. Oslo)

We review some potential applications of random Vandermonde matrices in the field of signal processing and wireless communications, and present new analytical methods for finding moments of such matrices. Random Vandermonde matrices play an important role in signal processing and communication applications such as direction of arrival estmation, precoding and sparse sampling theory. Using asymptotic results based on the theory of random Vandermonde matrices, we show through several application examples, such as deconvolution and wireless capacity analysis, the research potential of this theory. The asymptotic results turn out to be valid for dimensions which are of interest to the community. The simulations confirm that random matrix theory is a useful theory for understanding the behaviour of the eigenvalues of matrices.

“Capacity of Correlated MIMO Channels: Channel Power and Multipath Sparsity”
Akbar Sayeed (Univ. Wisconsin)

RMT has been instrumental in studying the capacity of multiple antenna (MIMO) wireless channels, especially for the i.i.d. channel model representing a rich multipath scattering environment. However, extension of the theory to spatially correlated channels has been challenging. In this talk, we present results on the capacity of MIMO channels based on the physically motivated notion of multipath sparsity as a source of correlation. Our first point of departure is the nature of channel normalization: all existing results assume that the channel power scales quadratically with the number of antennas, N, a legacy of the i.i.d. model, resulting in a linear O(N) capacity scaling. While this assumption may be justified for relatively small N, it violates power conservation principles for large N. We thus consider channels with an arbitrary sub-quadratic power scaling and argue that it necessarily leads to sparse MIMO channels in which the independent degrees of freedom (DoF) also scale at a sub-quadratic rate with N. We study the coherent capacity of sparse MIMO channels from two perspectives:1) capacity scaling with N, and 2) capacity as a function of SNR for a fixed N. Sparse channels afford a new dimension over which channel capacity can be optimized: the distribution of the DoF in the available spatial channel dimensions. Our investigation is based on a family of sparse channel configurations to which existing RMT results, with appropriate modifications, can be applied. We identify the needed modifications and show that the capacity of all channels in the family admits a simple and intuitive closed-form approximation that reveals a new tradeoff between the multiplexing gain and the received SNR. From a capacity scaling perspective, we identify an ideal channel configuration that yields the fastest (sub-linear) scaling with N. For fixed N, we show that the capacity-maximizing configuration depends on the transmit SNR and optimizes the multiplexing gain-received SNR tradeoff. Surprisingly, only three canonical configurations suffice for near-optimal performance over the entire SNR range. Different configurations can be realized in practice by adapting the antennas' spacings at the transmitter and the receiver.

Joint with Vasanthan Raghavan (UIUC).

"Eigenvalues of Large Dimensional Random Matrices"

Jack Silverstein (NC State)

We will outline recent work on spectral properties of random matrices. One topic to be covered concerns the properties of individual eigenvalues of a class of matrices of sample covariance type: $B_n = (1/N)T_n^{1/2}X_nX_n^*T_n^{1/2}$ where $X_n = (X_{ij})$ is $n \times N$ with i.i.d. complex standardized entries, and $T_n^{1/2}$ is a Hermitian square root of the nonnegative definite Hermitian matrix T_n. This matrix can be viewed as the sample covariance matrix of N i.i.d. samples of the n dimensional random vector $T^{1/2}(X_n)_{\cdot 1}$. It is known that if $n/N \to c > 0$ and the empirical distribution function (e.d.f.) of the eigenvalues of T_n converge as $n \to \infty$, then the e.d.f. of the eigenvalues of B_n converges a.s. to a nonrandom limit. This result is relevant in multivariate analysis where the vector dimension is large, but the number of samples to adequately approximate the population matrix (required in standard statistical procedures) cannot be attained.

If a finite number of eigenvalues of T_n are outside the support of its limiting spectral e.d.f, the model is referred to as "spiked". Results are obtained for the limiting behavior of those eigenvalues of B_n which correspond to the "spiked" eigenvalues of T_n. An application is given to the detection problem in array signal processing: determining the number of sources (presumed large) impinging on a bank of sensors in a noise filled environment (joint with Jinho Baik at University of Michigan, and with Raj Rao at MIT).

Another class takes the form $C_n = (1/N)(R_n + \sigma X_n)(R_n + \sigma X_n)^*$ where X_n is as in B_n, $\sigma > 0$, and R_n is $n \times N$ random, independent of X_n with the spectral e.d.f. of $(1/N)R_n R_n$ converging to a nonrandom limit. These matrices model situations in array signal processing, where information is contained in the sampling of the vectors $R_{\cdot 1} \cdots R_{\cdot N}$, but the received vector is contaminated by additive noise (the columns of σX_n). The e.d.f. of the eigenvalues of C_n also converges a.s. as $n \to \infty$ (with $n/N \to c > 0$). Properites of the limiting distribution will be outlined. (joint with Brent Dozier).

A third class to be discussed generalizes B_n. It is of the form $D_n = (1/N)T_n^{1/2} X_n S_n X_n^* T_n^{1/2}$ where S_n is $N \times N$ nonnegative definite Hermitian, and appears in the modeling of MIMO (multiple-input-multiple-out) systems in wireless communications (joint work with Debashis Paul at UC Davis).

"What is operator-valued free probability theory and why should an engineer care about it"

Roland Speicher (Queens University)

I will explain the operator-valued generalization of free probability theory in the context of random matrices. My main point will be that many asymptotic results or approximations of random matrices by calculable deterministic models (a la Girko) consist in replacing independent Gaussians by free semicirculars. Matrices of semicirculars are, in contrast to matrices of Gaussians, accessible to an exact analysis. If the variance of the semicirculars depends on the position in the matrix, then this exact analysis requires operator-valued free probability.

"Largest Eigenvalue Distributions and their Applications"

Craig A. Tracy (UC Davis)

This will be a survey talk on the distribution of largest eigenvalues arising in random matrix theory. These distribution functions, first appearing in the Gaussian ensembles, have subsequently been shown to have a "universal" character both in random matrix theory and other areas such as growth processes. These developments will be surveyed in this talk. The talk is not designed for experts in random matrix theory since the results discussed have been known for some time.

"Bridging the Gap: Free Probability and Channel Capacity"
Antonia Tulino (Princeton)

Free Probability is a recent mathematical theory which tries to understand non-commutative algebras by using techniques inspired by classical probability theory. This approach has been very successful, solving some longstanding problems in the field of operator algebras. Quite surprisingly, it has turned out that the methods and results of free probability can also be used to describe the spectral properties of random matrices. The latter appear in many models in applied sciences; e.g., Wishart random matrices are at the basis of modern statistics and similar kind of random matrices are used to model several classes of channels that arise in wireless communications. In the last few years, a considerable body of work has emerged in the communications and information theory literature on the fundamental limits of communication channels that makes substantial use of results in free probability and in random matrix theory.

The purpose of these talks is to illustrate the synergy between random matrix theory, free probability and information/communication theory. Specifically we will first give a general description of some of the main classical existing results of free probability and its connection with random matrix theory. In the second part we will address the applications of free probability and random matrix theory in information/communication problems. Furthermore, motivated by engineering problems we will point out some of the directions for extensions of the classical free probability results.

"The interplay between random matrices and information theory"
Sergio Verdu (Princeton)

In this talk I will give a brief introduction to those results on the spectrum of random matrices that are relevant to the analysis of the capacity of communication channels, and will review a few tools and results in random matrices inspired by information theory.