3510 - Solutions - Assignment 9

4.31 This guy parks for a total of two hours a day. Since the police "arrivals" form a Poisson process of rate α per hour, the number of police to come by while he is parked is a Poisson random variable of parameter 2α. Therefore, the probability he is ticketed is one minus the probability this Poisson takes value zero. That is, $1 - e^{-2\alpha}$.

4.34 For the first part of the question, we are looking at the arrivals of buses being a Poisson process of rate 5 per hour. Since each bus stays two hours, at 4pm the buses that we see are all those that arrived between 2pm and 4pm, a two hour window. Thus, the number of buses we see at 4pm is a Poisson random variable for which $\lambda = 2 \cdot 5 = 10$.

For the second part, there are two types of busses. Type A stays for one hour and arrives at rate 5/4 per hour. Type B stays two hours and arrives at rate 15/4 per hour. So, the number of Type A's we see at 4pm is a $Poisson(5/4)$ r.v. The number of Type B's we see at 4pm is a $Poisson(2 \cdot 15/4)$ per hour.

Since Type A's and Type B's arrive independently, the total number of busses has a Poisson distribution with parameter $\lambda = 5/4 + 2 \cdot 15/4 = 35/4$.

4.35 This is a conditional probability question:

$$P(\text{at least 15 illegally parked} \mid \text{75 total cars}) = 1 - \sum_{k=0}^{14} P(k \text{ cars illegally parked} \mid \text{75 total cars}).$$

For each term on the right hand side we have

$$\begin{aligned} P(k \text{ cars illegally parked} \mid \text{75 total cars}) &= \frac{P(k \text{ illegally parked})P(75-k \text{ legally parked})}{P(\text{75 total cars })} \\ &= \frac{e^{-5}\frac{5^k}{k!} \cdot e^{-45}\frac{45^{75-k}}{(75-k)!}}{e^{-50}\frac{50^75}{75!}} = \binom{75}{k}(9/10)^{75-k}(1/10)^k. \end{aligned}$$

You will notice that this is the mass function at k for a $Bin(75, 1/10)$ r.v. - we've discussed this in class. You can substitute this in the above and get a number, if you want.

5.2 The thing to remember here is that the margin of error in any confidence interval decays like $\frac{1}{\sqrt{n}}$ where n is the number of trials (or "people" "polled"). Here they report that with

500 trials they have an error of $\pm .021$ and with 1000 trials an error of $\pm .01$. The $1/\sqrt{n}$ law says to reduce your error by a factor of 2 you must do 4 times the number of trials. But here they are claiming that they have cut their error in half by only doubling the number of trials. Something fishy is going on.

5.3 You are being asked to compute $P(W < 550)$ where $W \sim N(799.5, 121.4^2)$. We know

$$P(W < 550) = P\left(Z < \frac{550 - 799.5}{121.4}\right) = P(Z < -2.055)$$

for Z a *standard* normal random variable. The table shows that this is about 0.02, so very rare.

5.21 The expected value of total rolls in 1000 rolls of a fair die is $1000 \cdot 3.5 = 3500$. The standard deviation is $\sqrt{1000 \cdot (35/12)} \approx 54$. (I used here that the variance of a single die roll is $35/12$).

This guy tells you he rolled a die 1000 times, and his total value was 3200. This is 300 less than the mean, which is to say well more than three standard deviations below the mean. 99% of the time, this won't happen so you can be "confident" that he wasn't using a fair die.