MATH 2001 - REVIEW FOR TEST 2

The test will cover Chapters 3 & 4 though you can safely ignore the description of the antipodal map from Section 4.1. I'll probably ask several definitions; below are sample (non-definition) exercises.

1. Let R_n be the number of ways to place n non-attacking rooks on an $n \times n$ chess-board.
(a) Prove that $|R_n| = n!$.
(b) Define $f : R_n \to \mathbb{Z}$ by

$$f(r) = \text{ number of rooks in } r \text{ on the diagonal squares}$$

(for $r \in R_n$). Is f injective? Is it surjective? Is there a partition of R_n described by f?

2. How many ways are there to write 10 as the sum of four non-negative numbers? (Here we count $0 + 0 + 1 + 9$ as distinct from $0 + 1 + 0 + 9$.) What if we insist that each of the four numbers are at least one?

3. A classroom has 14 students and two rows of seats with 8 seats in each row. If there must be at least 5 students in the first row and 4 students in the second row, how many possible ways are there to seat the students?

4. How many partitions of $\{1, 2, 3, \dots n\}$ have exactly two cells?

5. Write down a proof that the n-cube cannot be realized as a planar graph for any $n \geq 4$.

6. Using the correspondence with Dyck paths, prove that the Catalan numbers C_n satisfy

$$C_{n+1} = \sum_{k=0}^{n} C_k C_{n-k}$$

(with $C_0 = 1$).

7. Draw the tree with Prüfer code $(2, 4, 4, 1, 7)$.