

## REVIEW FOR TEST 1

The test will cover Chapters 1 & 2 though note we did not consider Gray Codes (from 1.3), Hamming Codes (from 1.4) the Collatz problem or shift registers (from 1.5), or Egyptian fractions (from 2.2).

Some sample problems:

1. Prove

$$\sum_{k=1}^n \frac{1}{4k^2 - 1} = \frac{n}{2n + 1}.$$

2. For any sets  $A$  and  $B$  prove that

$$P(A \cap B) = P(A) \cap P(B).$$

3. Use the Euclidean algorithm to show that  $\gcd(100, 21) = 1$  and to find integers  $a$  and  $b$  such that

$$1 = a \cdot 21 + b \cdot 100.$$

4. In  $\mathbb{Z}_{11}$  compute:

- i)  $[10]^{36} - 1$ ,
- ii)  $\sqrt{[3]}$ ,
- ii)  $[6]^{-1}$ .

5. Prove that the elements of the Fibonacci sequence satisfy:

- i)  $F_{2n} = F_{n+1}^2 - F_{n-1}^2$ ,
- ii)  $F_{n+1}F_{n-1} - F_n^2 = (-1)^n$ .

6. Explain why (in words)

$$\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2.$$