THIRD ASSIGNMENT

DUE MONDAY, OCTOBER 31

The following problems are in Chapter 15 of *Concise*, where there are some hints. Let $n \geq 1$ and π be an abelian group. In this problem, you will construct the Moore space $M\pi, n)$ and the Eilenberg-MacLane space $K(\pi, n)$. You can use the Hurewicz Theorem.

(1) Construct a connected CW-complex $M(\pi, n)$ such that

$$\widetilde{H}_*(M(\pi, n), \mathbb{Z}) = \begin{cases} \pi & * = n \\ 0 & \text{otherwise.} \end{cases}$$

(2) Give an example of an $M(\mathbb{Z}/2, 1)$ and of a $M(\mathbb{Z}, 2)$.

(3) (Chapter 15, Problem 4) Construct a connected CW-complex $K(\pi, n)$ such that

$$\pi_*(K(\pi, n), \mathbb{Z}) = \begin{cases} \pi & * = n \\ 0 & \text{otherwise.} \end{cases}$$

(4) Let X be another CW-complex X whose only non-zero homotopy group is $\pi_n X = \pi$. Construct a homotopy equivalence $K(\pi, n) \to X$. Conclude that $K(\pi, n)$'s are unique up to a weak homotopy equivalence.

(5) Exhibit a fibration $F \to E \to B$ where, up to weak homotopy equivalence, F is a $K(\pi, n-1)$, B is a $K(\pi, n)$ and E is contractible.

(6) (Optional) Let ρ be an abelian group. Prove that map $[K(\pi, n), K(\rho, n)]_* \to \mathrm{Hom}_{\mathbb{Z}}(\pi, \rho)$ which sends $f : K(\pi, n) \to K(\rho, n)$ to $\pi_n(f) : \pi \to \rho$ is an isomorphism.