SECOND ASSIGNMENT

DUE FRIDAY OCTOBER 7

(1) Suppose that $E \xrightarrow{p} B$ is a fibration of unbased spaces with B path connected. For $b \in B$, let $F_b = p^{-1}(b)$. Prove that F_{b_1} is homotopy equivalent to F_{b_2} for all $b_1, b_2 \in B$.

(2) Prove that there are homeomorphisms $\Sigma C_f \cong C_{\Sigma f} \cong C_{-\Sigma f}$, where C_f here denotes the reduced mapping cone.

(3) If $p : E \to B$ is a fibration and B is contractible, prove that there is a homotopy equivalence $\phi : E \to B \times F$ such that the following diagram commutes

$$\begin{tikzcd} E \arrow[rr, "\phi"] \arrow[dr, "p"'] & & B \times F \arrow[dl, "\pi_B"] \\ & B & \end{tikzcd}$$

(see 4.3.18 - Aguilar et al. for a hint.)

(4) (a) Compute $\pi_* S^1$.

(b) Use the Hopf fibration $S^1 \to S^3 \to S^2$ to compute $\pi_3 S^2$.

(c) Compute $\pi_* \mathbb{C}P^\infty$. (Hint: there are fiber bundles $S^{2n+1} \to \mathbb{C}P^n$ with fiber S^1. Can you write $\mathbb{C}P^\infty$ as the based space in a fiber bundle?).