

Mathematica is a powerful tool that can be used to carry out computations and construct graphs and images to help deepen our understanding of mathematical concepts. This document will serve as a living reference guide that you should continue to update as you learn Mathematica's syntax and functionality.

1. Basic Computations

To begin, we will focus on using Mathematica to evaluate basic algebraic expressions. Open a new document in Mathematica to produce a blank Notebook. To evaluate the expression

$$13^{-\frac{2}{3}} \times \sqrt[4]{65} + \sqrt{2}$$

in Mathematica, we enter in the Notebook the expression:

`13^(-2/3)*Surd[65,4]+Sqrt[2]`

To evaluate this expression, press shift+enter. Mathematica will provide the output:

$$\sqrt{2} + \frac{\sqrt[4]{5}}{13^{5/12}}$$

Now use Mathematica to evaluate the expression:

$$\frac{\log_2(64)}{\sqrt{50}}$$

Write your input and output from Mathematica:

Solution:

Mathematica Input:

`Log[2,64]/Sqrt[50]`

Mathematica Output:

$$\frac{3\sqrt{2}}{5}$$

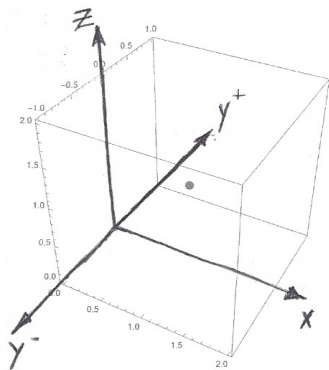
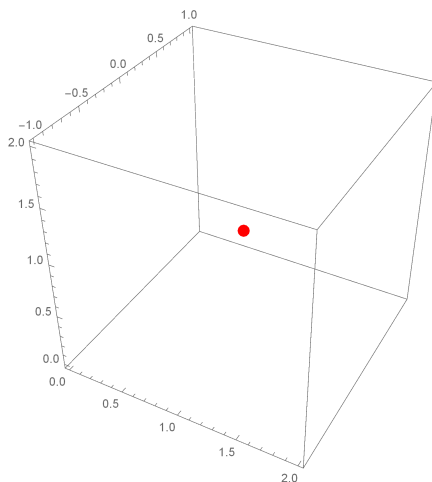
2. Plotting Points in 3-Space This semester, being able to visualize points, vectors, functions, and relations in 3-space will be essential. Let's start with plotting a point in 3-space.

(a) To plot the point, input the following in Mathematica:

```
Graphics3D[{Red, PointSize[0.03], Point[{1, 0, 1}]}, Axes -> True]
```

Confirm your output from Mathematica matches the picture below.

(b) Draw the coordinate axes on this output by hand using the axes detailed in the Guidelines for Graphing in 3-space. Observe that Mathematica has a different viewpoint of the axes. It will be important to be flexible moving between these two perspectives.



Solution:

3. Vector Operations

We can calculate vector operations (such as vector addition and scalar multiplication) using Mathematica. Before we can do this, we must use proper vector syntax in Mathematica. To define the vector

$$\mathbf{a} = \langle 1, e^{-2} \rangle$$

we input the following:

`a={1,E^(-2)}`

Given the vectors $\mathbf{r} = \langle 1, 4, 3 \rangle$ and $\mathbf{s} = \langle -2, 4, -6 \rangle$, compute the vector \mathbf{u} defined below:

$$\mathbf{u} = \frac{1}{\sqrt{(-1)^2 + (8)^2 + (-3)^2}}(\mathbf{r} + \mathbf{s})$$

- (a) Write your input and output from Mathematica.

Solution:

Mathematica Input:

`r={1,4,3}; s={-2,4,-6}`

`u=1/Sqrt[(-1)^2+(8)^2+(-3)^2]*(r+s)`

Mathematica Output:

$$\left\{ -\frac{1}{\sqrt{74}}, 4\frac{2}{\sqrt{37}}, -\frac{3}{\sqrt{74}} \right\}$$

- (b) Use the internet search Mathematica documentation to learn the Mathematica command for finding the magnitude of a vector. Use this to calculate the magnitude of the vector \mathbf{u} using Mathematica. Write your input and output from Mathematica.

Solution:

Mathematica Input:

`Norm[u]`

Mathematica Output:

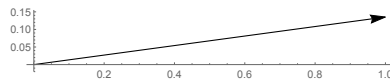
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4. Graphing Vectors

It can be useful to graph vectors to better understand how different vector operations work. We can graph the vector $\mathbf{a} = \langle 1, e^{-2} \rangle$ in 2-space by inputting the following into Mathematica:

```
Graphics[{Arrow[{{0, 0}, {1, E^(-2)}}]}, Axes -> True]
```

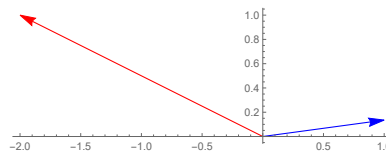
We get the output graph:



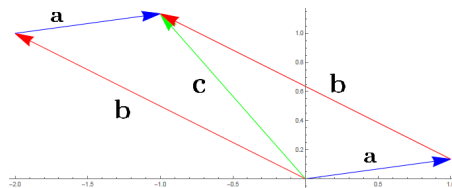
- (a) To graph the two vectors $\mathbf{a} = \langle 1, e^{-2} \rangle$ and $\mathbf{b} = \langle -2, 1 \rangle$ on the same axes in 2-space we input:

```
Graphics[{Blue, Arrow[{{0, 0}, {1, E^(-2)}}]},
          Red, Arrow[{{0, 0}, {-2, 1}}]}, Axes->True]
```

We get the output graph:



Use Mathematica to graph the vector $\mathbf{c} = \mathbf{a} + \mathbf{b}$ on the same axes as \mathbf{a} and \mathbf{b} , and draw the output below. Describe the relationship you observe between these three vectors using your output.

**Solution:**

Observe that if we add the head of the vector \mathbf{b} to the tail of the vector \mathbf{a} (or the head of the vector \mathbf{a} to the tail of the vector \mathbf{b}) we have the resulting vector $\mathbf{c} = \mathbf{a} + \mathbf{b}$.

- (b) Write your input for graphing the vector $\mathbf{d} = -2\mathbf{a}$ on the same axes as \mathbf{a} . Describe the relationship you observe between these two vectors using your output.

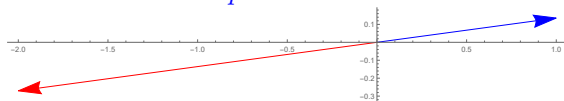
Solution:

Mathematica Input:

```
a={1,E^(-2)}; d=-2*a;
```

```
Show[{Graphics[{Blue, Arrow[{{0, 0}, a}]}], {Graphics[{Red, Arrow[{{0, 0}, d}]}]}, Axes -> True]
```

Mathematica Output:



Notice that the vector \mathbf{d} is in the opposite direction of \mathbf{a} and double in length. Multiplying by a scalar not equal to 1 results in a scaling of our original vector. Multiplying by negative scalar changes the direction of the vector.

5. Graphing Functions

While it is useful to use Mathematica to compute and simplify complicated expressions, we can also use Mathematica to help us visualize relations and functions.

- (a) To graph a half-circle in rectangular coordinates with the function

$$f(x) = \sqrt{1 - x^2},$$

input the following in Mathematica, confirming it is a half-circle.

```
Plot[Sqrt[1 - x^2], {x, -1, 1}, AspectRatio -> Automatic]
```

- (b) Find a polar equation, $r = g(\theta)$, whose graph is the same as in part (a). Be sure to give the domain for your independent variable.

Solution: $r = 1$, for $0 \leq \theta \leq \pi$

Confirm you are correct by graphing your polar equation in Mathematica by using PolarPlot. Search the internet to find the appropriate syntax.

- (c) We can also describe this same graph by defining a parametrized curve in the xy -plane. Write a parametric equation that traces this half-circle starting on the positive x -axis.

Solution:

$$\mathbf{r}(t) = \langle t, \sqrt{1 - t^2} \rangle, \text{ for } -1 \leq t \leq 1 \text{ or}$$

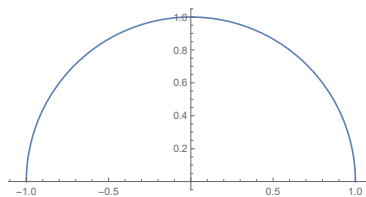
$$\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle, \text{ for } 0 \leq t \leq \pi$$

both work.

To confirm these are correct we can use the *Mathematica Input*:

```
ParametricPlot[{t, Sqrt[1 - t^2]}, {t, -1, 1}] or ParametricPlot[{Cos[t], Sin[t]}, {t, 0, Pi}]
```

to result in the *Mathematica Output*:



Confirm you are correct by graphing your parametrized planar curve in Mathematica by using ParametricPlot. Search the internet to find the appropriate syntax. Observe that Mathematica does not indicate the direction of your curve.

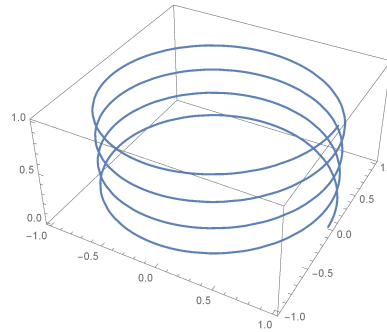
- (d) Now create a parametrization for the same half-circle that traces the curve in the opposite direction of your previous parametrized curve.

Solution:

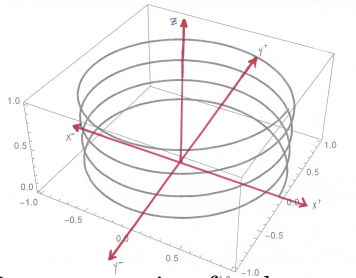
$$\mathbf{r}(t) = \langle -t, \sqrt{1 - t^2} \rangle, \text{ for } -1 \leq t \leq 1 \quad \text{or} \quad \mathbf{r}(t) = \langle -\cos(t), \sin(t) \rangle, \text{ for } 0 \leq t \leq \pi$$

6. We can use Mathematica to graph parametrized space curves.

- (a) Write a parametric function and the Mathematica input needed to produce the following helix in Mathematica. Draw the coordinate axes on this output by hand using the axes detailed in the Guidelines for Graphing in 3-space. *Note: The helix goes through the point (1,0,0) and (1,0,1).*

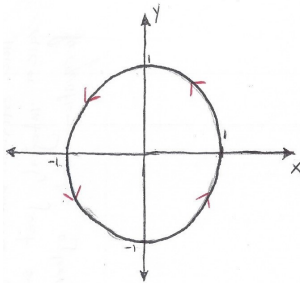


Solution: *Mathematica Input:*
`ParametricPlot3D[{Cos[t], Sin[t], t/{8*Pi}}, {t, 0, 8*Pi}]`



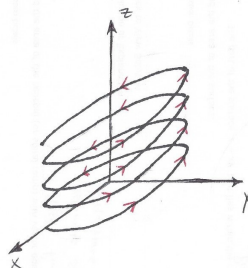
- (b) Draw a top view for the graph of your parametrized space curve.

Solution: Here is a sketch of the top view of the helix. Note the inclusion of path direction arrows.



- (c) Draw the helix you parameterized using the axes outlined in the Guidelines for Graphing in 3-space.

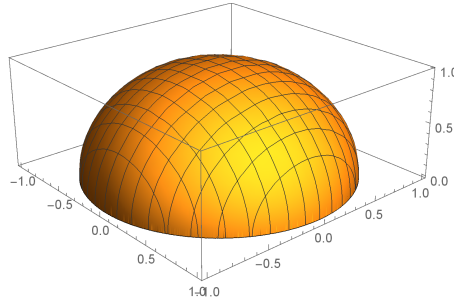
Solution: Here is a sketch of the helix. Note the inclusion of path direction arrows.



- (d) Imagine rotating the helix by 90 degrees about the z -axis so that it begins at the point $(0, 1, 0)$. Write a parametric equation for the resulting curve. Let the domain for the parameter t be $[0, 8\pi]$.

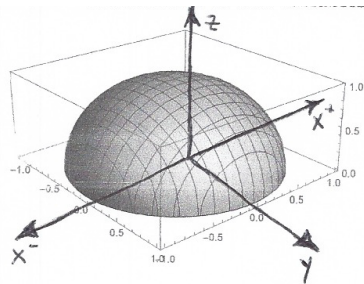
Solution: $\mathbf{r}(t) = \left\langle -\sin(t), \cos(t), \frac{t}{8\pi} \right\rangle$, for $0 \leq t \leq 8\pi$

7. We can also create graphical representations for functions of two variables in 3-space, such as this hemisphere.



Again, observe that Mathematica uses a different axes viewpoint than described in the Guidelines for Graphing in 3-Space.

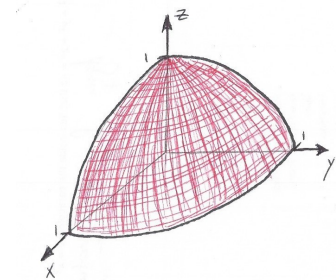
- (a) Draw the xyz -axes on the above graph by hand.



Solution:

Note that x -axis and y -axis are interchangeable due to symmetry of hemisphere.

- (b) Now graph this hemisphere on your own axes by hand.



Solution: Here is a sketch of the hemisphere in the first octant.

- (c) Recall that the equation for a circle of radius 1 centered at the origin is given by $x^2 + y^2 = 1$. Use this idea to think about a possible equation for a sphere of radius 1 centered at the origin. Now write the function that describes the graph of the given hemisphere.

Solution: Since the equation for a circle in 2-space is given by $x^2 + y^2 = 1$, we can infer that 3-space we will need to include the additional depth variable z in our equation for a sphere. The equation for a sphere of radius 1 centered at the origin is given by $x^2 + y^2 + z^2 = 1$. To create a function we need to ensure that for each (x, y) in the plane that there is only one associated z value.

Connecting back to the idea of the circle, we could create a function for the top half of the circle with $y = \sqrt{1 - x^2}$. Therefore, we can create the function for the hemisphere:

$$z = \sqrt{1 - x^2 - y^2}$$

- (d) Recreate this graph using Mathematica by using Plot3D. Search the internet for the appropriate syntax.

Solution: *Mathematica Input:* `Plot3D[Sqrt[1 - x^2 - y^2], {x, -1, 1}, {y, -1, 1}]`