

Consider the following functions:

$$h(x) = e^x$$

$$f(x, y) = x^2 + y^2$$

$$\mathbf{g}(x, y) = \langle x + y, 3x - y, 2x + y \rangle$$

$$\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$$

$$\mathbf{p}(t) = \langle -\sin t, \cos t \rangle$$

$$\mathbf{w}(x, y, z) = \langle 2x, 2y \rangle$$

For each of the compositions below, indicate whether or not they are defined by circling (a) or (b). If they are defined, fill in the boxes to show the dimensions of the input and output spaces.

1. (a) $f \circ h : \mathbb{R}^{\square} \mapsto \mathbb{R}^{\square}$
 (b) $f \circ h$ is not defined.
2. (a) $h \circ f : \mathbb{R}^2 \mapsto \mathbb{R}^1$
 (b) $h \circ f$ is not defined.
3. (a) $\mathbf{g} \circ \mathbf{w} : \mathbb{R}^3 \mapsto \mathbb{R}^3$
 (b) $\mathbf{g} \circ \mathbf{w}$ is not defined.
4. (a) $\mathbf{w} \circ \mathbf{g} : \mathbb{R}^2 \mapsto \mathbb{R}^2$
 (b) $\mathbf{w} \circ \mathbf{g}$ is not defined.
5. (a) $f \circ \mathbf{g} : \mathbb{R}^{\square} \mapsto \mathbb{R}^{\square}$
 (b) $f \circ \mathbf{g}$ is not defined.
6. (a) $\mathbf{g} \circ \mathbf{p} : \mathbb{R}^1 \mapsto \mathbb{R}^3$
 (b) $\mathbf{g} \circ \mathbf{p}$ is not defined.
7. (a) $\mathbf{r} \circ \mathbf{w} : \mathbb{R}^{\square} \mapsto \mathbb{R}^{\square}$
 (b) $\mathbf{r} \circ \mathbf{w}$ is not defined.
8. (a) $f \circ \mathbf{p} : \mathbb{R}^1 \mapsto \mathbb{R}^1$
 (b) $f \circ \mathbf{p}$ is not defined.
9. There are 6 more meaningful compositions of the given functions that were not named above. List at least three of them.

Solution: The remaining legitimate compositions are $\mathbf{p} \circ f : \mathbb{R}^2 \mapsto \mathbb{R}^2$, $\mathbf{r} \circ f : \mathbb{R}^2 \mapsto \mathbb{R}^3$, $\mathbf{w} \circ \mathbf{r} : \mathbb{R}^1 \mapsto \mathbb{R}^2$, $f \circ \mathbf{w} : \mathbb{R}^3 \mapsto \mathbb{R}^1$, $\mathbf{p} \circ h : \mathbb{R} \mapsto \mathbb{R}^2$, and $\mathbf{r} \circ h : \mathbb{R} \mapsto \mathbb{R}^3$.