Math 2400, Final Exam

May 7, 2019

PRINT YOUR NAME: ____________________

PRINT INSTRUCTOR'S NAME: ____________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50 AM
☐	Section 002	Harrison Stalvey	8:00–8:50 AM
☐	Section 003	Daniel Martin	9:00–9:50 AM
☐	Section 004	Albert Bronstein	9:00–9:50 AM
☐	Section 005	Xingzhou Yang	10:00–10:50 AM
☐	Section 006	Mark Pullins	10:00–10:50 AM
☐	Section 007	János Englander	10:00–10:50 AM
☐	Section 008	John Willis	12:00–12:50 PM
☐	Section 009	Taylor Klotz	1:00–1:50 PM
☐	Section 010	János Englander	2:00–2:50 PM
☐	Section 011	Harrison Stalvey	2:00–2:50 PM
☐	Section 012	Xingzhou Yang	3:00–3:50 PM
☐	Section 013	Trevor Jack	4:00–4:50 PM

Question	Points	Score
1	8	
2	10	
3	3	
4	10	
5	10	
6	10	
7	10	
8	10	
9	3	
10	10	
11	10	
12	6	
Total:	100	

Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 150 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like **100/7** or expressions like **ln(3)/2** as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (8 points) **Note: No partial credit for this problem.**

Let $\vec{\boldsymbol{a}} = \langle \mathbf{2},\ \mathbf{-2},\ \mathbf{1} \rangle$, $\vec{\boldsymbol{b}} = \langle \mathbf{1},\ \mathbf{-2},\ \mathbf{3} \rangle$. Compute

(a) $|\vec{\boldsymbol{a}}| =$ ________________

(b) $\mathbf{2}\vec{\boldsymbol{a}} - \mathbf{3}\vec{\boldsymbol{b}} =$ ________________

(c) $\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} =$ ________________

(d) $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} =$ ________________

2. (10 points) Determine if the following function is **continuous** at the origin. Explain your reasoning.

$$f(x,y) = \begin{cases} \dfrac{x-y}{x+y} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

3. (3 points) Let $\boldsymbol{D}$ be a region in the $\boldsymbol{xy}$-plane with area $\boldsymbol{A}$ and boundary curve $\boldsymbol{C}$.

Which of the following vector fields will make $\displaystyle\int_C \vec{\boldsymbol{F}} \cdot \mathbf{d}\vec{\boldsymbol{r}} = \boldsymbol{A}$? Circle your answer.

(A) $\vec{\boldsymbol{F}} = \langle \boldsymbol{x} + \boldsymbol{y},\ \boldsymbol{x} \rangle$

(B) $\vec{\boldsymbol{F}} = \dfrac{\mathbf{1}}{\mathbf{2}} \langle \boldsymbol{x},\ \boldsymbol{x} - \boldsymbol{y} \rangle$

(C) $\vec{\boldsymbol{F}} = \left\langle \boldsymbol{y}^{\mathbf{2}} - \boldsymbol{y} + \mathbf{cos}\, \boldsymbol{x}^{\mathbf{2}},\ \mathbf{2}\boldsymbol{xy} \right\rangle$

(D) $\vec{\boldsymbol{F}} = \langle -\boldsymbol{y},\ \mathbf{2}\boldsymbol{x} \rangle$

4. (10 points) Given three points $\boldsymbol{A(1,-1,1)}$, $\boldsymbol{B(1,2,1)}$, $\boldsymbol{C(2,1,2)}$,

(a) Determine if the points are on the same line.

(b) Find the equation of a plane containing all three points.

5. (10 points) Let $\vec{\boldsymbol{F}} = \langle \boldsymbol{-2xy},\ \boldsymbol{y},\ \boldsymbol{z} \rangle$. Evaluate the line integral

$$\int_C \vec{\boldsymbol{F}} \cdot \mathrm{d}\vec{\boldsymbol{r}}$$

where $\boldsymbol{C}$ is given by $\vec{\boldsymbol{r}}(\boldsymbol{t}) = \langle \boldsymbol{t},\ \cos(2\boldsymbol{t}),\ \boldsymbol{t} + \sin(2\boldsymbol{t}) \rangle$ for $\boldsymbol{0} \leq \boldsymbol{t} \leq \boldsymbol{\pi}$.

6. (10 points) $\boldsymbol{S}$ is the part of the paraboloid $\boldsymbol{z = x^2 + y^2}$ that lies between the planes $\boldsymbol{z = 1}$ and $\boldsymbol{z = 4}$.

(a) Parametrize the surface $\boldsymbol{S}$. Include the bounds for the parameter(s).

(b) Use your parameterization from part (a) to rewrite the integral with appropriate bounds. You need not evaluate this rewritten integral.

$$\iint_{\boldsymbol{S}} \boldsymbol{z}\, \mathbf{d}\boldsymbol{S}.$$

7. (10 points) Use the **Method of Lagrange Multipliers** to find the maximum and minimum values of the function

$$\boldsymbol{f(x, y) = x + y}$$

under the constraint $\boldsymbol{g(x, y) = 2x^2 + 2xy + y^2 = 4}$.

8. (10 points) Let $\boldsymbol{S}$ be the part of the hemisphere $\boldsymbol{x^2 + y^2 + z^2 = 4}$ above the $\boldsymbol{xy}$-plane, with positive (outward) orientation and let

$$\vec{\boldsymbol{F}} = \left\langle \boldsymbol{xz \sin(yz) + x^3,\ \cos(yz),\ 3zy^2 + z^3 - 7} \right\rangle .$$

Find $\displaystyle\iint_{\boldsymbol{S}} \vec{\boldsymbol{F}} \cdot \mathbf{d}\vec{\boldsymbol{S}}$ using the **Divergence Theorem**. **Hint:** $\boldsymbol{S}$ is **not** a closed surface.

Warning: No credit if the Divergence Theorem is not used!

9. (3 points) Let $\boldsymbol{f(x, y, z) = x^2 + 2y^2 + 3z^2}$. Find an equation of the tangent plane to the level surface $\boldsymbol{x^2 + 2y^2 + 3z^2 = 30}$ at the point $\boldsymbol{(1,\ -1,\ 3)}$. Circle the correct answer:

(A) $\boldsymbol{2x - 4y + 18z = 0}$

(B) $\boldsymbol{2x + 4y + 18z = 0}$

(C) $\boldsymbol{2(x - 1) - 4(y + 1) + 18(z - 3) = 0}$

(D) $\boldsymbol{2(x + 1) - 4(y - 1) + 18(z + 3) = 0}$

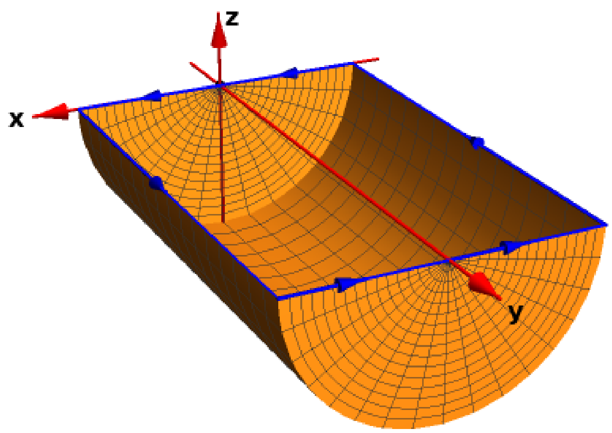
10. (10 points) Evaluate $\oint_C \vec{F} \cdot \mathbf{d}\vec{r}$ where

$$\vec{F}(x, y, z) = \left\langle e^{x^2} + 2y,\ x - \sin^2 y,\ e^x \cos y + \sin(e^z) \right\rangle$$

and C is the boundary of the surface consisting of the half cylinder given by

$$\vec{r}(u, v) = \langle \cos u,\ v,\ -\sin u \rangle$$

where $0 \leq u \leq \pi$, $0 \leq v \leq 4$, along with the two semi-circle caps at $y = 0$ and $y = 4$. C is oriented counter-clockwise viewed from top (the positive z direction).



11. (10 points) Let $\vec{F}$ be a vector field on $\mathbb{R}^3$ and f a scalar function on $\mathbb{R}^3$ such that each have continuous third order derivatives.

Determine if the following are always, sometimes, or never true.

(a) If $f(x, y, z)$ is a scalar function then $\nabla \times (\nabla \times (\nabla f)) = \vec{0}$.

(A) Always true
(B) Sometimes true
(C) Never true

(b) If $\vec{F}(x, y, z)$ is any vector field then $\nabla \cdot (\nabla \times \vec{F}) = 0$.

(A) Always true
(B) Sometimes true
(C) Never true

(c) $\operatorname{div}(\operatorname{grad} f) = 0$.

(A) Always true
(B) Sometimes true
(C) Never true

(d) $\operatorname{div}(\operatorname{curl}(\operatorname{curl}(\operatorname{curl} \vec{F}))) = \langle 0, 0, 0 \rangle$.

(A) Always true
(B) Sometimes true
(C) Never true

(e) $\nabla \cdot (f\vec{F}) = \nabla f \cdot \vec{F} - f \, \nabla \cdot \vec{F}$, where f and $\operatorname{div}\vec{F}$ are not zero.

(A) Always true
(B) Sometimes true
(C) Never true

12. (6 points) Suppose that f is a differentiable function of x and y, and

$$g(u,v) = f(u^2 - v^3,\ u + 2v).$$

	$f(x,y)$	$g(u,v)$	$f_x(x,y)$	$f_y(x,y)$
$(3,4)$	2	-1	8	6
$(2,1)$	-2	2	3	1

(a) Use the table of values to compute $g_u(2,1)$.

(A) -6

(B) 7

(C) 13

(D) 14

(E) 38

(b) If $g_v(1,-1) = 5$ and $f_x(2,-1) = 1$, compute $f_y(2,-1)$.

(A) 3

(B) 4

(C) 5

(D) 6

(E) 7