Math 2400, Midterm 2

March 11, 2019

PRINT your name: ______________________________

PRINT instructor's name: ______________________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50 AM
☐	Section 002	Harrison Stalvey	8:00–8:50 AM
☐	Section 003	Daniel Martin	9:00–9:50 AM
☐	Section 004	Albert Bronstein	9:00–9:50 AM
☐	Section 005	Xingzhou Yang	10:00–10:50 AM
☐	Section 006	Mark Pullins	10:00–10:50 AM
☐	Section 007	János Englander	10:00–10:50 AM
☐	Section 008	John Willis	12:00–12:50 PM
☐	Section 009	Taylor Klotz	1:00–1:50 PM
☐	Section 010	János Englander	2:00–2:50 PM
☐	Section 011	Harrison Stalvey	2:00–2:50 PM
☐	Section 012	Xingzhou Yang	3:00–3:50 PM
☐	Section 013	Trevor Jack	4:00–4:50 PM

Question	Points	Score
1	8	
2	5	
3	5	
4	10	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	8	
12	8	
13	8	
Total:	100	

Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 95 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like $100/7$ or expressions like $\ln(3)/2$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (8 points) **Note: No partial credit for this problem.**

   Let $f(x, y, z) = x^2y + e^{2z}$. Compute the following:

   (a) $\dfrac{\partial f}{\partial x} = $ ______________.

   (b) $\dfrac{\partial^2 f}{\partial z^2} = $ ______________.

   (c) $\dfrac{\partial^2 f}{\partial x \partial y} - \dfrac{\partial^2 f}{\partial y \partial x} = $ ______________.

   (d) $\nabla f(1, -1, 0) = $ ______________.

2. (5 points) Circle the answer that best describes each statement.

(a) If f is defined at $(0,0)$ and $\lim_{(x,y)\to(0,0)} f(x,y) = 0$, then f is continuous at $(0,0)$.

(A) Always true

(B) Sometimes true

(C) Never true

(b) If $f(x,y) \to L$ as $(x,y) \to (0,0)$ along every straight line through $(0,0)$, then $\lim_{(x,y)\to(0,0)} f(x,y) = L$.

(A) Always true

(B) Sometimes true

(C) Never true

(c) If $z = f(x,y)$ is a function of two variables and $\nabla f(a,b) = \langle 0,0\rangle$, then f has a local maximum or minimum at (a,b)

(A) Always true

(B) Sometimes true

(C) Never true

(d) If $f_x(x,y) = 0$ and $f_y(x,y) = 0$ for all (x,y), then f is constant.

(A) Always true

(B) Sometimes true

(C) Never true

(e) If $f(x,y)$ is a differentiable function and $\vec{u}$ is a unit vector, then the directional derivative $D_{\vec{u}} f(a,b)$ is parallel to $\vec{u}$.

(A) Always true

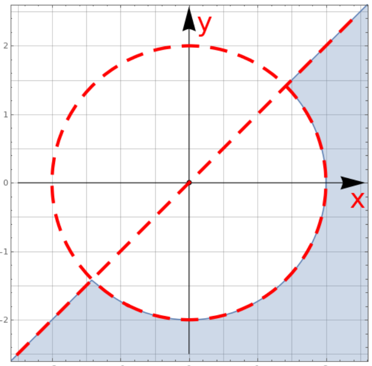
(B) Sometimes true

(C) Never true

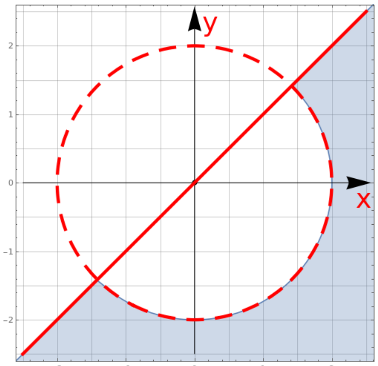
3. (5 points) Let $f(x,y) = \dfrac{\ln(x-y)}{\sqrt{4-x^2-y^2}}$.

Which one of the following shaded regions is the domain of f? __________.

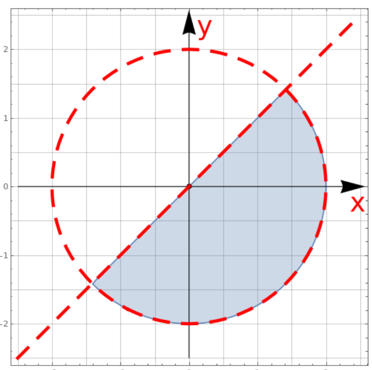
(A)



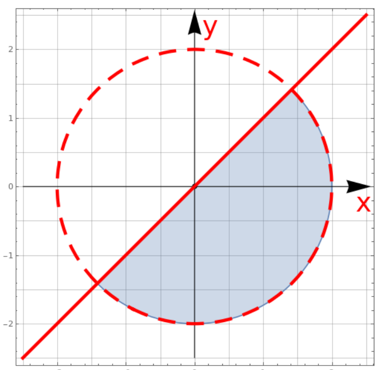
(E)



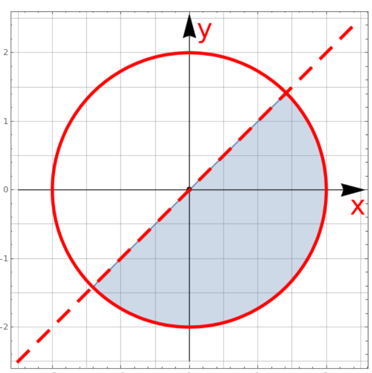
(B)



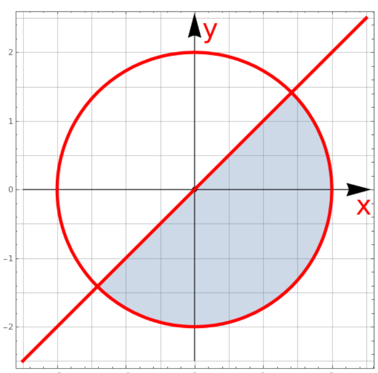
(F)



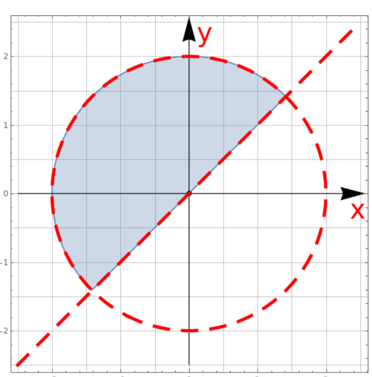
(C)



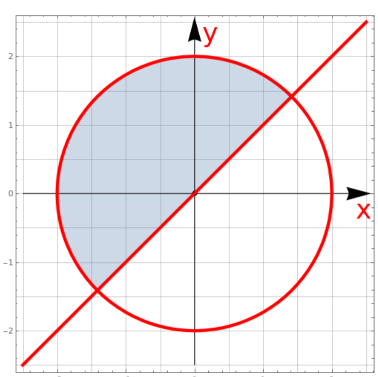
(G)



(D)

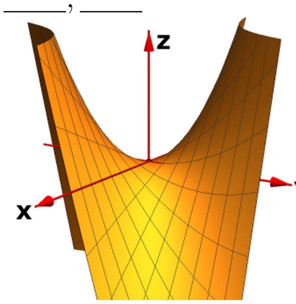


(H)

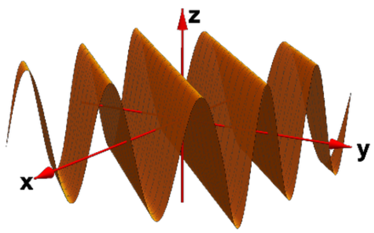


4. (10 points) Match each 3D surface with one of the contour plots, and one of the equations.

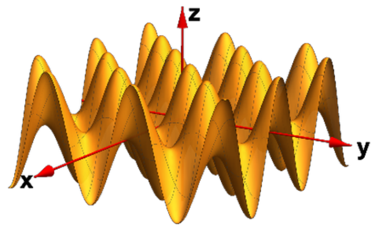
(a) ____, ____



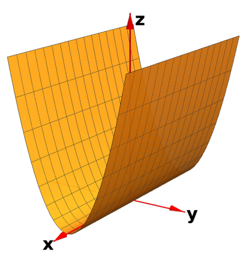
(b) ____, ____



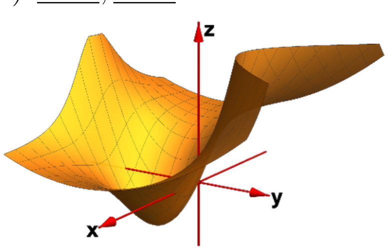
(c) ____, ____



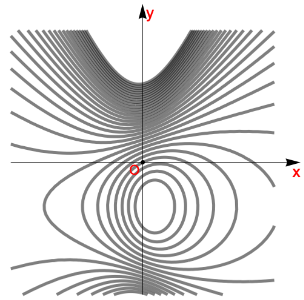
(d) ____, ____



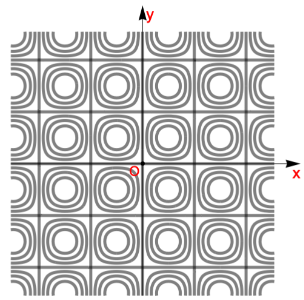
(e) ____, ____



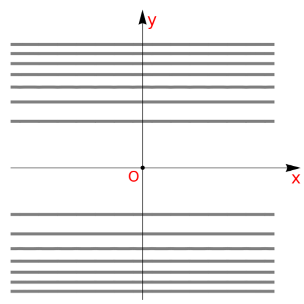
(1)



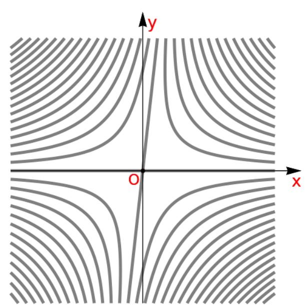
(2)



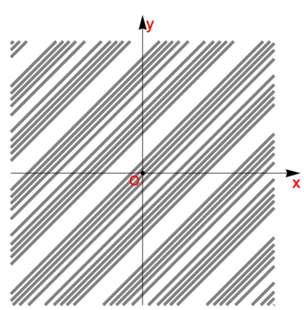
(3)



(4)



(5)



(A) $z = \sin(x)\sin(y)$

(B) $z = \sin(x - y)$

(C) $z = \dfrac{x^2 - x + y^2 + 2y}{x^2 + 1}$

(D) $z = y^2$

(E) $z = y^2 - 9xy$

5. (8 points) Let S be the surface given by $z = xy$, and let R be the rectangle $[0, 6] \times [0, 4]$.

(a) Taking the sample points to be the upper right corners, use a **Riemann sum** with $m = 2$ and $n = 2$ to estimate the volume of the solid that lies below S and above R.

(b) Calculate the **exact** volume of the solid that lies below S and above R.

6. (8 points) Let f be a differentiable function, $g(x, y) = f(u, v)$, where $u = x^2 - y^2$, $v = y^2 - x^3$.

$g(1, 2) = 11$	$g(-3, 3) = 7$	$f(1, 2) = 20$	$f(-3, 3) = 11$
$f_u(1, 2) = 4$	$f_v(1, 2) = 5$	$f_u(-3, 3) = 2$	$f_v(-3, 3) = -1$

Evaluate $g_x(1, 2)$ based on the values in the above table.

7. (8 points) A surface is represented by $z = f(x, y)$ with f differentiable. Let $\vec{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$. Suppose that $f_x(2, 1) = 5$ and $D_{\vec{u}} f(2, 1) = 3\sqrt{2}$.

(a) Find $f_y(2, 1)$.

(b) Assume $f(2, 1) = 3$. Use the result in (a) to find a vector in the xy-plane that is tangent to the level curve $f(x, y) = 3$ at the point $(2, 1)$.

8. (8 points) Find all critical points of the function

$$f(x, y) = \frac{2}{3}x^3 + 2x^2 + y^2 - 2xy$$

and classify each as a local maximum, local minimum, saddle point, or not enough information.

9. (8 points) Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - 3x^2y + y^2}{4x^2 + 4y^2}$ if it exists, and then prove it. Otherwise, explain why it does not exist.

10. (8 points) Find the absolute maximum and absolute minimum values of the function

$$f(x, y) = 4x + 4y - x^2 - y^2$$

subject to the constraint $x^2 + y^2 = 2$.

11. (8 points) Let S be a surface given by

$$\vec{r}(u, v) = \left\langle u\cos(v),\, u\sin(v),\, \ln\left(9 + u^2\right)\right\rangle$$

where $(u, v) \in [0, 2] \times [0, 2\pi)$. Find an equation of the tangent plane to S at the point $(0,\, 1,\, \ln(10))$.

12. (8 points) Evaluate the integral $\displaystyle\iint_D 6x^2y^2\,dA$, where D is the region bounded by $x = y^2$ and $y = x^3$ in the first quadrant.

13. (8 points) Evaluate the iterated integral by reversing the order of integration

$$\int_0^1 \int_{x^{1/3}}^1 \frac{1}{y^4+1}\, dy\, dx$$