1. Multiple Choice: For the following multiple choice questions, no partial credit is given. **Fill in your answer on the bubble sheet.**

(1) (3 points) **Fill in your answer on the bubble sheet.**
Suppose $f(x, y, z) = x + y^2 + z^2$, and let S be the level surface $f(x, y, z) = 8$. Find the equation of the tangent plane to S at the point $(-2, 1, 3)$.

(A) $(x + 2) + 2(y - 1) + 6(z - 3) = 0$

(B) $(x + 2) + 2(y - 1) + 2(z - 3) = 0$

(C) $(x - 2) + 2(y + 1) + 6(z + 3) = 0$

(D) $(x - 2) + 2(y + 1) + 6(z + 3) = 8$

(E) $(x - 2) + 2(y + 1) + 6(z + 8) = 0$

(F) $(x + 2) + 2(y - 1) + 6(z - 3) = 8$

(2) (3 points) **Fill in your answer on the bubble sheet.**
Find the parametrization of the part of the elliptic paraboloid $y = 4x^2 + z^2 - 4$ that lies inside the cylinder $x^2 + z^2 = 4$.

(A) $\langle x,\ 4x^2 + z^2 - 4,\ z\rangle$ for $-1 \le x \le 1$ and $-2 \le z \le 2$

(B) $\langle x,\ x^2 + z^2,\ z\rangle$ for $-2 \le x \le 2$ and $0 \le z \le 4$

(C) $\langle x,\ 4 - x^2 - z^2,\ z\rangle$ for $-2 \le x \le 2$ and $-\sqrt{4 - x^2} \le z \le \sqrt{4 - x^2}$

(D) $\langle r\cos\theta,\ r^2 + 3r^2\cos^2\theta - 4,\ r\sin\theta\rangle$ for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$

(E) $\langle r\cos\theta,\ r^2 - 4,\ 2r\sin\theta\rangle$ for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$

(F) $\left\langle \frac{1}{2}r\cos\theta,\ r^2 - 4,\ r\sin\theta\right\rangle$ for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$

(3) (3 points) **Fill in your answer on the bubble sheet.**

Suppose

$$f(x, y) = ye^{-x} + 3x.$$

Find the direction of the maximum rate of increase of $f(x, y)$ at $(0, 1)$.

(A) $\langle 2, 1\rangle$

(B) $\langle -2, -1\rangle$

(C) $\langle 3, 0\rangle$

(D) $\langle -3, 0\rangle$

(E) $\langle 2e^{-1}, e\rangle$

(F) $\langle -2e^{-1}, -e\rangle$

(4) (3 points) **Fill in your answer on the bubble sheet.**

Find the following limit, if it exists.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - 2y^2}{x^2 + y^2}$$

(A) 0

(B) 1

(C) -1

(D) 2

(E) -2

(F) The limit does not exist.

(5) (3 points) **Fill in your answer on the bubble sheet.**
Let

$$f(x,y) = \begin{cases} \dfrac{x+2}{x^2+y^2+1}, & \text{if } (x,y) \neq (0,0) \\ a & \text{if } (x,y) = (0,0) \end{cases}$$

Find a, such that the function $f(x,y)$ is continuous at $(0,0)$.

(A) 0

(B) 1

(C) -1

(D) 2

(E) -2

(F) There is no a for which f is continuous at (0,0).

(6) (3 points) **Fill in your answer on the bubble sheet.**
Let

$$f(x,y) = (x^3 - x)(y^2 - 1).$$

Find $f_{xy}(x,y)$.

(A) $(x^3 - x)(2y)$

(B) $(3x^2 - 1)(2y)$

(C) $(3x^2 - 1)(y^2 - 1) + (x^3 - x)(2y)$

(D) $(3x^2 - 1)(y^2 - 1)$

(E) 0

(F) $6x^2y + 2y - 3x^2 - 1$

(7) (3 points) **Fill in your answer on the bubble sheet.**

Let S be the surface parametrized by $\vec{r}(\theta, z) = \langle 3\cos(\theta), 3\sin(\theta), z\rangle$, for $0 \leq \theta \leq 2\pi$ and $0 \leq z \leq 2$. Evaluate

$$\iint_S 1\, dS.$$

(A) π

(B) 2π

(C) 3π

(D) 9π

(E) 12π

(F) 18π

(8) (3 points) **Fill in your answer on the bubble sheet.**

Let

$$\vec{F}(x, y, z) = \langle xyz, \quad xy + yz + zx, \quad x + y + zy\rangle.$$

Find curl$\vec{F}$.

(A) $\langle yz, \quad x + z, \quad 1\rangle$

(B) $\langle 1 - x - y + z, \quad -1 + xy, \quad y + z - xz\rangle$

(C) $\langle 1 + x + y, \quad xy, \quad xz\rangle$

(D) $\langle yz + y + z + 1, \quad xz + x + z + 1, \quad xy + x + y + 1\rangle$

(E) $\langle 1, \quad 1, \quad 1\rangle$

(F) $\langle y + z, \quad x + z, \quad x + y\rangle$

(9) (3 points) **Fill in your answer on the bubble sheet.**

Let

$$\vec{F}(x,y,z) = \left\langle x^3 + y^2,\ ze^{-y},\ x^2\sin(z) \right\rangle.$$

Find div$\vec{F}$

(A) $\left\langle 3x^2, -e^{-y}z, x^2\cos(z) \right\rangle$

(B) $\left\langle -e^{-y},\ -2x\sin(z),\ -2y \right\rangle$

(C) $\left\langle 3x^2 + 2y,\ -e^{-y},\ 2x\cos(z) \right\rangle$

(D) $3x^2 - ze^{-y} + x^2\cos(z)$

(E) $3x^2 + 2y - e^{-y} + 2x\cos(z)$

(F) $3x^2 + ze^{-y} - x^2\cos(z)$

(10) (3 points) **Fill in your answer on the bubble sheet.**

Let f be a scalar-valued function of three variables and $\vec{F}$ a vector field on $\mathbb{R}^3$. Which of the following must be true for all such f and $\vec{F}$? (Assume all functions and their components are polynomials.)

(A) div(div f) $= 0$

(B) div(grad f) $= 0$

(C) curl(div f) $= 0$

(D) div(curl(curl $\vec{F}$)) $= 0$

(E) curl(curl(div $\vec{F}$)) $= 0$

(F) grad(curl $\vec{F}$) $= 0$

2. (7 points) Convert the following integral from rectangular coordinates to cylindrical coordinates. Fill in all **7** blanks.

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{18-2x^2-2y^2} yz \; dz \; dy \; dx$$

$$= \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{} \; dz \; dr \; d\theta$$

3. (7 points) Convert the following integral from spherical coordinates to rectangular coordinates. Fill in all **7** blanks.

$$\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{2\sqrt{2}} \sin\phi \;\, d\rho \, d\phi \, d\theta$$

$$= \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{} \; dz \; dy \; dx$$

4. (10 points) Evaluate the integral

$$\int_C \left(xy^2 + y\right)\, dx + \left(2x^2y + e^{y^2}\right)\, dy$$

where C is boundary of the rectangle in the xy-plane oriented **clockwise** with vertices $(0,0)$, $(0,3)$, $(2,3)$, and $(2,0)$.

5. (8 points) For the following function, find all local maximums, local minimums, and saddle points.

$$f(x, y) = x^4 - 2x^2 + y^3 - 3y$$

6. (5 points) Consider the vector field $\vec{F}$ on $\mathbb{R}^2$ given by

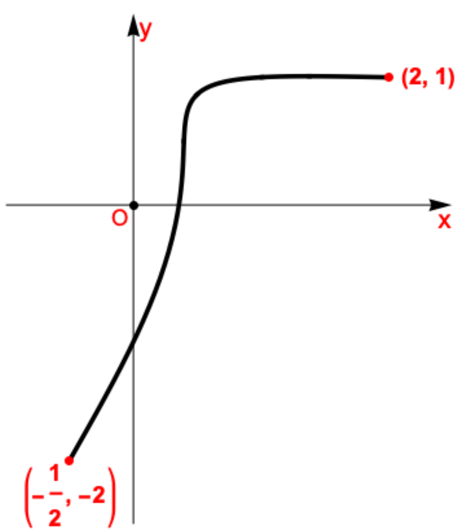
$$\vec{F}(x, y) = \langle \pi \cos(\pi x) + y,\ x + 2y \rangle .$$

Find a potential function $f(x, y)$ for $\vec{F}(x, y)$ such that $\nabla f = \vec{F}$.

7. (3 points) Let $\vec{F} = \nabla g$ where $g(x, y) = e^{\cos(\pi x)} + xy$. Evaluate the integral

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is the path pictured below from $\left(-\frac{1}{2}, -2\right)$ to $(2, 1)$.



8. (10 points) Let S be the helicoid parameterized by

$$\vec{r}(u,v) = \langle u\sin v,\ 2v,\ u\cos v\rangle \quad \text{for } 0 \le u \le 1, \quad 0 \le v \le \pi,$$

oriented in the direction of the positive y-axis. Let $\vec{F}$ be a vector field given by

$$\vec{F} = xy\vec{i} + (y^2+1)\vec{j} + yz\vec{k}.$$

Evaluate $\displaystyle\iint_S \vec{F} \cdot d\vec{S}$.

9. (10 points) Let $\vec{F}$ be a vector field on $\mathbb{R}^3$ given by

$$\vec{F} = (\cos x + y)\vec{i} + (e^y + xz^2)\vec{j} + (2z^2 + yx)\vec{k}.$$

Let C be a circle of radius 1 centered at $(0, 0, 2)$ lying on the plane $z = 2$, which is oriented **counterclockwise** when viewed from above. Evaluate $\int_C \vec{F} \cdot d\vec{r}$.

10. (10 points) Let

$$\vec{F}(x,y,z) = \left(x^3 + e^{y^2+z^2}\right)\vec{i} + \left(\cos(x^4) + y^3\right)\vec{j} + \left(\ln(x^2+4) + z^3\right)\vec{k}$$

be a vector field on $\mathbb{R}^3$, region E be the part of the solid sphere $x^2+y^2+z^2 \leq 4$ in the **first octant**, and S be the boundary of E oriented outward. Find the total flux of $\vec{F}$ through S:

$$\iint_S \vec{F} \cdot d\vec{S}.$$