1. (3 points) Evaluate the following limit. You do not need to show any work.

$$\lim_{(x,y)\to(1,-1)} e^{-xy}\cos(x+y)$$

(A) 0

(B) $\dfrac{1}{e}$

(C) $e\pi$

(D) e

(E) 1

(F) Does not exist

2. (5 points) Consider the following integral

$$\int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx$$

Select the answer that is equivalent to the integral above. You do not need to show any work.

(A) $\displaystyle\int_0^2 \int_{\frac{y}{2}}^{\sqrt{y}} f(x,y)\,dx\,dy$

(B) $\displaystyle\int_0^4 \int_{\frac{y}{2}}^{\sqrt{y}} f(x,y)\,dx\,dy$

(C) $\displaystyle\int_{\frac{y}{2}}^{\sqrt{y}} \int_0^2 f(x,y)\,dx\,dy$

(D) $\displaystyle\int_0^4 \int_{\sqrt{y}}^{\frac{y}{2}} f(x,y)\,dx\,dy$

(E) $\displaystyle\int_0^2 \int_{\sqrt{y}}^{\frac{y}{2}} f(x,y)\,dx\,dy$

3. Let $f(x, y) = x^3 + e^{xy}$.

(i) (4 points) Evaluate $\nabla f(-1, 3)$.

(ii) (4 points) What is the value of the directional derivative of f at the point $P(-1, 3)$ in the direction of $\vec{v} = \langle -2, 5 \rangle$?

4. Suppose that g is a differentiable function of x and y, and

$$f(u,v) = g(x,\, y), \text{ where } x = u^2 - 2v,\ y = 3u - v.$$

Suppose that the following facts are given:

	$g(x,y)$	$f(u,v)$	$g_x(x,y)$	$g_y(x,y)$
$(1,0)$	4	3	4	5
$(1,3)$	3	5	-1	2

(i) (2 points) What is the geometric meaning of the partial derivative g_x?

(ii) (5 points) Find $f_u(1,0)$.

5. (13 points) Consider the function $f(x, y) = x^3 + 2y^2 - 3x - 4y$. Find all critical points and classify each as a local maximum, a local minimum, or a saddle point or say there is not enough information.

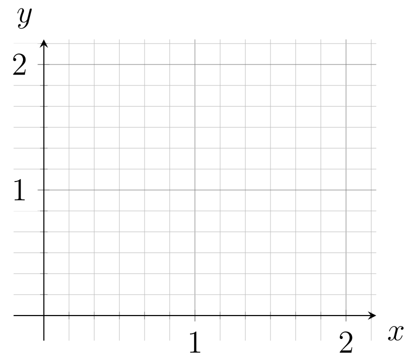
6. (12 points) Consider the parametric surface $\vec{r}(s,t) = \langle s + 3t,\, st^2 + s,\, \ln(t)\rangle$. Find an equation for the tangent plane at $P(1, -4, 0)$.

7. (12 points) Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^6}{(x^2+y^2)^4}$$

8. Consider the integral $\displaystyle\iint_D 5x\,dA$, where D is the region bounded by the y-axis, the curves $y + x = 2$ and $y - x^2 = 0$ in the first quadrant.

(i) (4 points) Sketch and shade the region D. Label any intersection points.



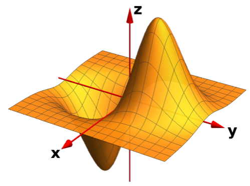
(ii) (3 points) What does $\displaystyle\iint_D 5x\,dA$ represent geometrically?

(iii) (8 points) Evaluate the integral.

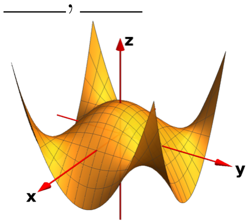
9. (13 points) Use **the method of Lagrange multipliers** to find the maximum and minimum values of the function $f(x, y) = xy$ subject to the constraint $3x^2 + y^2 = 6$.

10. (12 points) Match each 3D surface with one of the contour plots, and one of the equations. Not all the equations will be matched.

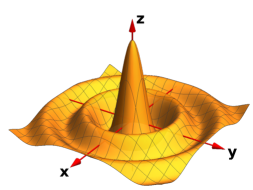
(i) _____, _____



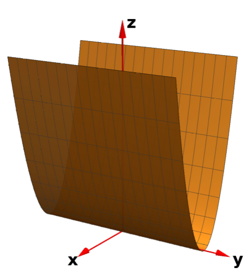
(ii) _____, _____



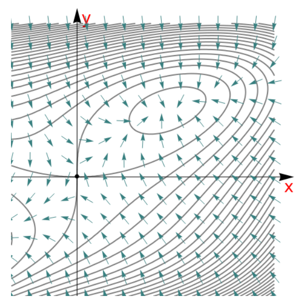
(iii) _____, _____



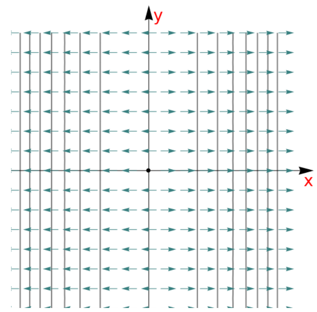
(iv) _____, _____



(1)



(2)



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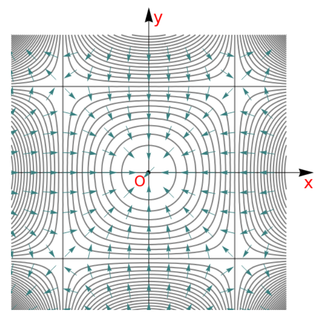
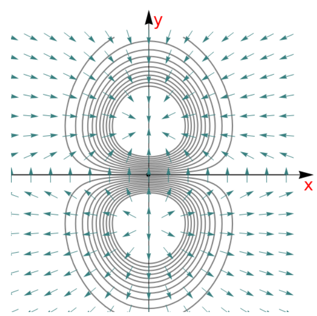
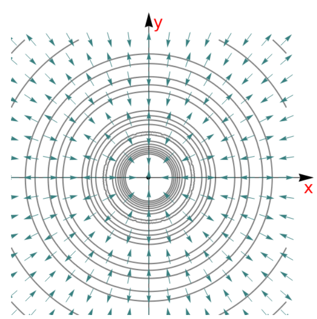


(4)



(5)



(A) $z = x^2$

(B) $z = y^2$

(C) $z = x^2 + y^2$

(D) $z = (1 - x^2)(1 - y^2)$

(E) $z = ye^{-2x^2 - y^2}$

(F) $z = \dfrac{\sin\left(\sqrt{x^2 + y^2}\right)}{\sqrt{x^2 + y^2}}$

(G) $z = \sin(xy)$

(H) $z = \sin x \sin y$