Math 2400, Midterm 3

April 16, 2018

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50
☐	Section 002	Xingzhou Yang	8:00–8:50
☐	Section 003	Albert Bronstein	9:00–9:50
☐	Section 004	Cliff Blakestad	10:00–10:50
☐	Section 005	Albert Bronstein	10:00–10:50
☐	Section 006	Mark Pullins	11:00–11:50
☐	Section 009	Taylor Klotz	11:00–11:50
☐	Section 007	Albert Bronstein	12:00–12:50
☐	Section 008	Martin Walter	1:00–1:50
☐	Section 010	Braden Balentine	2:00–2:50
☐	Section 011	Pedro Berrizbeitia	3:00–3:50
☐	Section 012	Pedro Berrizbeitia	4:00–4:50

Question	Points	Score
1	12	
2	12	
3	8	
4	12	
5	16	
6	12	
7	12	
8	16	
Total:	100	

Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like $\mathbf{100/7}$ or expressions like $\mathbf{\ln(3)/2}$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (12 points) Match the vector fields $\vec{F}$ with the plots below.

(1) $\vec{F} = \langle y, x \rangle$ ____

(2) $\vec{F} = \langle x, y \rangle$ ____

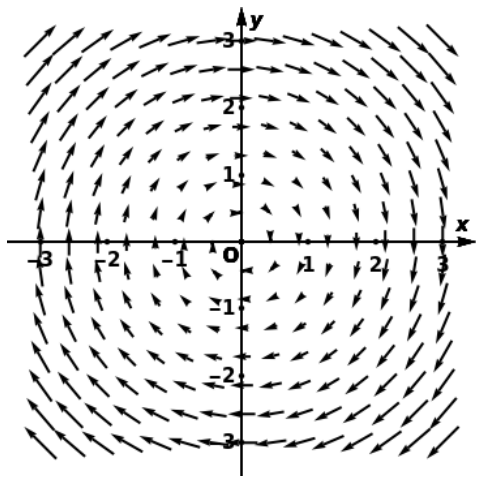
(3) $\vec{F} = \langle y, -x \rangle$ ____

(4) $\vec{F} = \langle x, -y \rangle$ ____

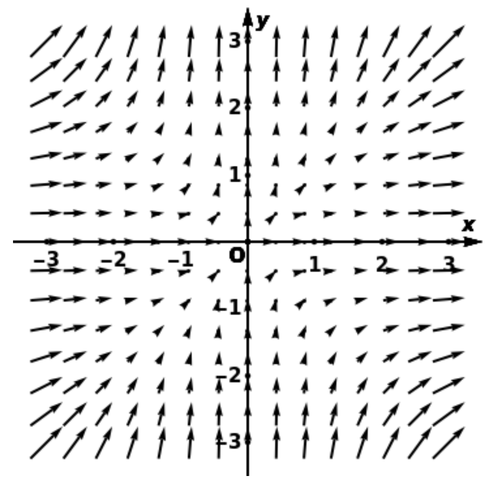
(5) $\vec{F} = \langle x^2, y^2 \rangle$ ____

(6) $\vec{F} = \langle 2 - x, 1 - y \rangle$ ____

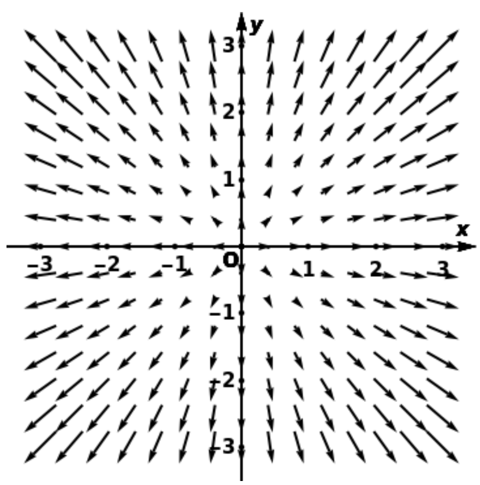
(A)



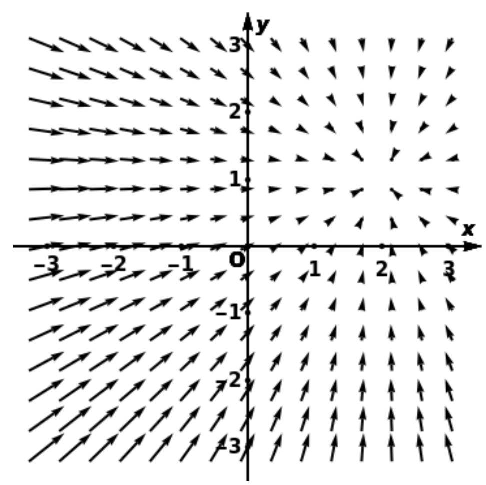
(D)



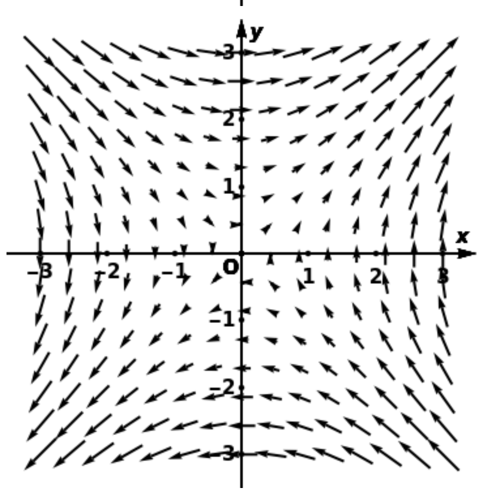
(B)



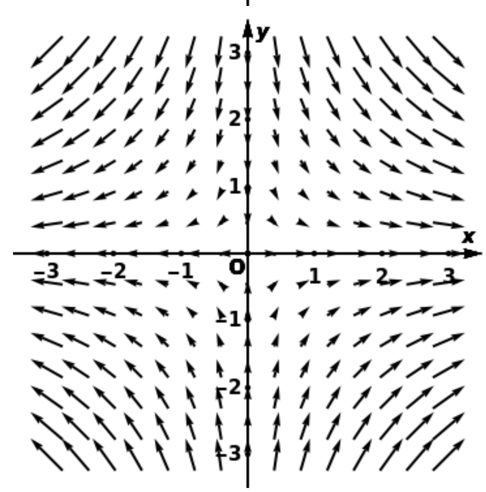
(E)



(C)



(F)



2. Consider the transformation $\boldsymbol{T^{-1}: R \to S}$ given by $\boldsymbol{T^{-1}(x, y) = (y, y - x^2)}$, where $\boldsymbol{R}$ is the region in the first quadrant of the $\boldsymbol{xy}$-plane bounded by $\boldsymbol{y = 0}$, $\boldsymbol{y = 16}$, $\boldsymbol{y = x^2}$, and $\boldsymbol{y = x^2 - 9}$.

(a) (4 points) If $\boldsymbol{u = y}$ and $\boldsymbol{v = y - x^2}$, find the transformation $\boldsymbol{T(u, v)}$ from $\boldsymbol{S}$ to $\boldsymbol{R}$.

(b) (4 points) Calculate the Jacobian, $\boldsymbol{\dfrac{\partial(x, y)}{\partial(u, v)}}$, of the transformation $\boldsymbol{T}$.

(c) (4 points) Use transformation $\boldsymbol{T}$ to evaluate $\displaystyle\iint_{\boldsymbol{R}} \boldsymbol{x\,dA.}$

3. (8 points) Select the integral that is ALWAYS equivalent to the integral given. No work is required.

(1) $\displaystyle\int_0^1 \int_{y^2+2}^{\sqrt{y}+2} \int_0^{x+y} f(x,y,z)\,dz\,dx\,dy$

(A) $\displaystyle\int_0^1 \int_{y^2+2}^{\sqrt{y}+2} \int_0^{x+y} f(x,y,z)\,dz\,dy\,dx$

(B) $\displaystyle\int_0^1 \int_{\sqrt{x-2}}^{(x-2)^2} \int_0^{x+y} f(x,y,z)\,dz\,dy\,dx$

(C) $\displaystyle\int_2^3 \int_{\sqrt{x-2}}^{(x-2)^2} \int_0^{x+y} f(x,y,z)\,dz\,dy\,dx$

(D) $\displaystyle\int_2^3 \int_{(x-2)^2}^{\sqrt{x-2}} \int_0^{x+y} f(x,y,z)\,dz\,dy\,dx$

(2) $\displaystyle\int_0^1 \int_0^{x} \int_0^{y} f(x,y,z)\,dz\,dy\,dx$

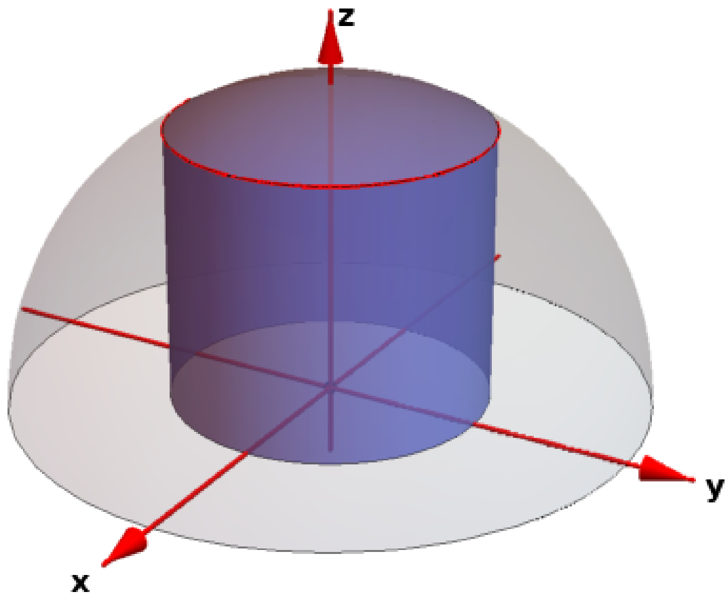
(A) $\displaystyle\int_0^1 \int_1^{z} \int_1^{x} f(x,y,z)\,dy\,dx\,dz$

(B) $\displaystyle\int_0^1 \int_z^{1} \int_x^{1} f(x,y,z)\,dy\,dx\,dz$

(C) $\displaystyle\int_0^1 \int_0^{1} \int_x^{z} f(x,y,z)\,dy\,dx\,dz$

(D) $\displaystyle\int_0^1 \int_z^{1} \int_z^{x} f(x,y,z)\,dy\,dx\,dz$

4. The solid $\boldsymbol{E}$ is inside the cylinder $\boldsymbol{x^2+y^2 = 1}$, below the sphere $\boldsymbol{x^2+y^2+z^2 = 4}$, and above the plane $\boldsymbol{z = 0}$. The density of the solid is $\boldsymbol{\rho(x, y, z) = \sqrt{x^2 + y^2}}$.



(a) (6 points) Set up, but do **not** evaluate, the integral for the mass of the solid.

(b) (6 points) If $\boldsymbol{(\bar{x},\ \bar{y},\ \bar{z})}$ is the center of mass of the solid, set up, but do **not** evaluate, the integral for $\boldsymbol{\bar{z}}$.

5. Let $\boldsymbol{S}$ be the parametric surface $\vec{\boldsymbol{r}}(\boldsymbol{u},\boldsymbol{v}) = \langle \boldsymbol{u}\cos(\boldsymbol{v}), \boldsymbol{u}\sin(\boldsymbol{v}), \boldsymbol{v}\rangle$ for $\boldsymbol{0} \le \boldsymbol{u} \le \sqrt{\boldsymbol{2}}$ and $\boldsymbol{0} \le \boldsymbol{v} \le \boldsymbol{u}$.

(a) (8 points) Compute $\vec{\boldsymbol{r}}_{\boldsymbol{u}} \times \vec{\boldsymbol{r}}_{\boldsymbol{v}}$.

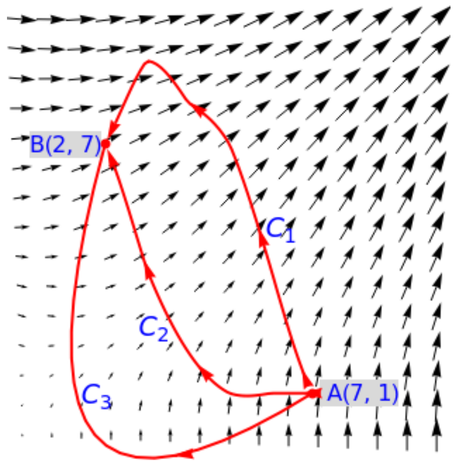
(b) (8 points) Determine the surface area of $\boldsymbol{S}$.

6. (12 points) Evaluate $\int_C (x^2 + y^2 + z^2)\, ds$ where

$$C: \; x = \sqrt{8}\, t, \;\; y = \cos t, \;\; z = \sin t, \;\; 0 \le t \le 1.$$

7. (12 points) Let $\vec{F}(x, y, z) = \langle y, x + e^z, ye^z + 1\rangle$. Calculate the integral $\int_C \vec{F} \cdot d\vec{r}$ where C is given by $\vec{r}(t) = \langle t^2, t^3, \sin(\pi t)\rangle$ for $0 \leq t \leq 2$.

8. The figure below shows the vector field $\vec{\boldsymbol{F}}(\boldsymbol{x}, \boldsymbol{y}) = \langle \boldsymbol{y}, \boldsymbol{x} + \mathbf{cos}\, \boldsymbol{y} \rangle$, and 3 curves $\boldsymbol{C_1}$, $\boldsymbol{C_2}$, $\boldsymbol{C_3}$ start at $\boldsymbol{A(7, 1)}$ and end at $\boldsymbol{B(2, 7)}$.



(a) (4 points) Determine if $\vec{\boldsymbol{F}}$ is conservative. Justify.

(b) (4 points) Explain why $\displaystyle\int_{\boldsymbol{C}} \vec{\boldsymbol{F}} \cdot \mathbf{d}\vec{\boldsymbol{r}}$ has the same value for all three paths.

(c) (8 points) Evaluate $\displaystyle\int_{\boldsymbol{C}} \vec{\boldsymbol{F}} \cdot \mathbf{d}\vec{\boldsymbol{r}}$.