Math 2400, Final Exam

December 18, 2018

PRINT YOUR NAME: ____________________

PRINT INSTRUCTOR'S NAME: ____________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50
☐	Section 002	Philip Kopel	8:00–8:50
☐	Section 003	Daniel Martin	8:00–8:50
☐	Section 004	Albert Bronstein	9:00–9:50
☐	Section 005	Mark Pullins	9:00–9:50
☐	Section 006	Xingzhou Yang	9:00–9:50
☐	Section 007	Martin Walter	10:00–10:50
☐	Section 008	Kevin Manley	10:00–10:50
☐	Section 009	Albert Bronstein	1:00–1:50
☐	Section 010	Martin Walter	1:00–1:50
☐	Section 011	Xingzhou Yang	2:00–2:50
☐	Section 012	Taylor Klotz	2:00–2:50
☐	Section 013	Xingzhou Yang	3:00–3:50
☐	Section 014	Braden Balentine	4:00–4:50
☐	Section 015	Caroline Matson	4:00–4:50

Question	Points	Score
1	12	
2	9	
3	7	
4	9	
5	6	
6	9	
7	8	
8	8	
9	8	
10	8	
11	8	
12	8	
Total:	100	

Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 150 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like **100/7** or expressions like **ln(3)/2** as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (12 points) **Note: No partial credit for this problem.**

Let $\vec{\boldsymbol{a}} = \langle \mathbf{-1,\ 2,\ 2} \rangle$, $\vec{\boldsymbol{b}} = \langle \mathbf{3,\ -2,\ 1} \rangle$. Compute

(a) $|\vec{\boldsymbol{a}}| =$ ________________

(b) $\mathbf{-2}\vec{\boldsymbol{a}} + \mathbf{3}\vec{\boldsymbol{b}} =$ ________________

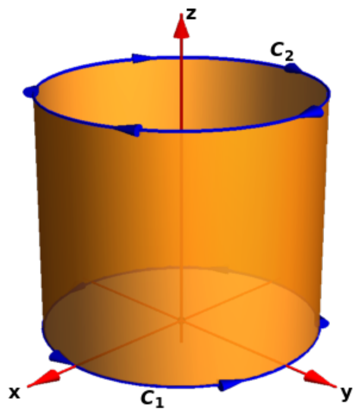
(c) $\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} =$ ________________

(d) $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} =$ ________________

2. (9 points) Let $\boldsymbol{\nabla} \times \vec{\boldsymbol{F}} = \langle \boldsymbol{3x, 3y, -6z} \rangle$ and let $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ be the circles of **radius two** centered on the $\boldsymbol{z}$-axis at $\boldsymbol{z = 0}$ and $\boldsymbol{z = 5}$, respectively. Calculate

$$\oint_{C_1} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} + \oint_{C_2} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$$

$\boldsymbol{C_1}$ is oriented counter-clockwise, and $\boldsymbol{C_2}$ is oriented clockwise viewed from the positive $\boldsymbol{z}$-axis.



3. (7 points) Find the distance between the two planes.

$$\boldsymbol{2x - y + 2z = 7}$$
$$\boldsymbol{2x - y + 2z = 1}$$

4. (9 points) Let $\boldsymbol{S}$ denote the surface given by the parameterization

$$\vec{r}(u, v) = (u^2 + v^2)\vec{i} + (u^2 - v^2)\vec{j} + uv\,\vec{k}, \qquad u \text{ is in } \mathbb{R}, \text{ and } v \geq 0.$$

Find an equation for the tangent plane to $\boldsymbol{S}$ at the point $\boldsymbol{P(1, -1, 0)}$.

5. (6 points) There will be **NO partial credit** awarded on the following questions – so be sure to double check your work!

(a) Suppose the amount of snow on the ground is given by the function $\boldsymbol{f(x,y) = x^4 + x^2y^2}$. If you are standing at the point $\boldsymbol{P(1,3)}$, in which direction would you walk to decrease how much snow you are standing in in the fastest?

(b) Let $\vec{\boldsymbol{F}} = (\boldsymbol{ye^{xy} + y^2})\vec{\boldsymbol{\imath}} + (\boldsymbol{xe^{xy} + x^2})\vec{\boldsymbol{\jmath}}$. Is $\vec{\boldsymbol{F}}$ conservative? If so, find the associated potential function.

(c) Suppose that $\boldsymbol{C}$ is a simple, smooth, positively-oriented curve which encloses the region $\boldsymbol{R}$ in the $\boldsymbol{xy}$-plane. If $\displaystyle\int_C \boldsymbol{5x\,dx + 3x\,dy = 30}$, what is the area of $\boldsymbol{R}$?

6. (9 points) Use the **Method of Lagrange Multipliers** to find all extrema of

$$\boldsymbol{f(x, y) = x^2 + y^2 - 2x - 4y}$$

constrained to the circle $\boldsymbol{x^2 + y^2 = 5}$.

7. (8 points) Suppose that $\boldsymbol{f}$ is a differentiable function of $\boldsymbol{x}$ and $\boldsymbol{y}$, and

$$\boldsymbol{g(s,t) = f(s^2 - 3t, 4s - t).}$$

	$f(x,y)$	$g(s,t)$	$f_x(x,y)$	$f_y(x,y)$
$(-2,3)$	5	1	2	3
$(1,1)$	4	5	6	7

(a) Use the table of values to compute $\boldsymbol{g_s(1,1)}$.

(b) If $\boldsymbol{g_t(0,-1) = 3}$ and $\boldsymbol{f_x(3,1) = 2}$, use the table of values to compute $\boldsymbol{f_y(3,1)}$.

8. (8 points) Consider the function

$$\boldsymbol{f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 2 & \text{if } (x,y) = (0,0) \end{cases}}$$

(a) Determine if the limit $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ exists. **Justify**.

(b) Determine if f is continuous at $(0, 0)$. **Justify**.

9. (8 points) Suppose the surface $\boldsymbol{S}$ of a small island with lizards is given by

$$\boldsymbol{z = 3e^{-x^2-y^2}}$$

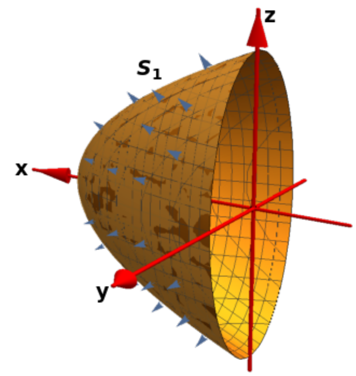
with $\boldsymbol{x^2 + y^2 \leq 100}$ and with all distances measured in miles. The population density of the lizards at a point $\boldsymbol{(x, y, z)}$ on the island is given by

$$\boldsymbol{\rho(x, y, z) = \frac{50}{1 + x^2 + y^2}}$$

lizards **per square mile**. Set up but **do not evaluate** an integral giving the total population of the lizards on the island.

$$\int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{} \, d\boxed{} \, d\boxed{}$$

10. (8 points) Let $\boldsymbol{S_1}$ be the part of the paraboloid $\boldsymbol{x = 1 - y^2 - z^2}$, oriented outward with $\boldsymbol{x \geq 0}$, and $\boldsymbol{E}$ be the solid enclosed by $\boldsymbol{S_1}$ and the $\boldsymbol{yz}$-plane. Let $\boldsymbol{S}$ be the boundary (closed surface) of $\boldsymbol{E}$, and



$$\vec{F} = \left\langle 2 + x\left(y^2 + z^2\right),\, x^3 + y^3,\, \sin(x^2) + z^3 \right\rangle.$$

(a) Use the **Divergence Theorem** to evaluate the flux through the surface $\boldsymbol{S}$, oriented outward.

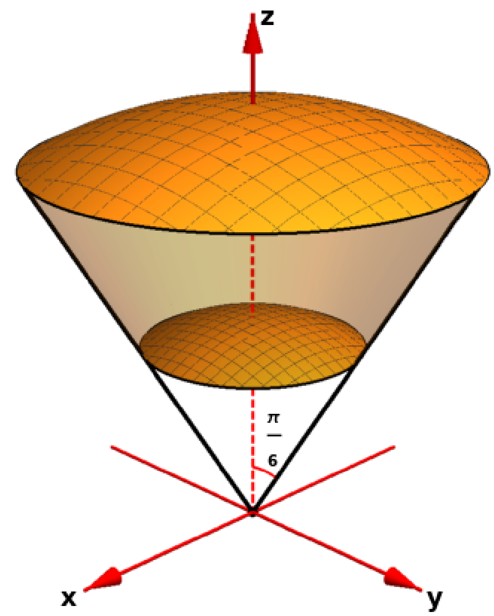
$$\iint_S \vec{F} \cdot d\vec{S}$$

(b) Evaluate the flux $\displaystyle\iint_{S_1} \boldsymbol{F} \cdot d\vec{S}$.

Hint: $\boldsymbol{S_1}$ is **NOT** a closed surface, and you may use the result in (a).

11. (8 points) Find the volume of the solid bounded by

$$\boldsymbol{x^2 + y^2 + z^2 = 4,}$$
$$\boldsymbol{x^2 + y^2 + z^2 = 1,} \quad \text{and}$$
$$\boldsymbol{z = \sqrt{3}\sqrt{x^2 + y^2}.}$$



12. (8 points) Below are a series of statements concerning gradient, curl and divergence. Assume that $\boldsymbol{f}$, $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{R}$ are scalar functions, $\vec{\boldsymbol{F}}$ is a vector field in $\mathbb{R}^{\mathbf{3}}$. If all of the second order partial derivatives exist and are continuous, circle the answer that best describes each statement.

(a) $\mathbf{curl}(\mathbf{grad}\,\boldsymbol{f}) = \vec{\mathbf{0}}$.

(A) Always true

(B) Sometimes true

(C) Never true

(b) $\mathbf{div}(\mathbf{curl}\,\vec{\boldsymbol{F}}) = \mathbf{0}$.

(A) Always true

(B) Sometimes true

(C) Never true

(c) $\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \boldsymbol{f}) = \mathbf{0}$.

(A) Always true

(B) Sometimes true

(C) Never true

(d) If $\vec{\boldsymbol{F}}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \langle \boldsymbol{P}(\boldsymbol{x}, \boldsymbol{y}),\ \boldsymbol{Q}(\boldsymbol{y}),\ \boldsymbol{R}(\boldsymbol{x}, \boldsymbol{z}) \rangle$, then $\mathbf{curl}\,\vec{\boldsymbol{F}}$ is orthogonal to the $\boldsymbol{x}$-axis.

(A) Always true

(B) Sometimes true

(C) Never true