Math 2400, Midterm 2

October 22, 2018

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50
☐	Section 002	Philip Kopel	8:00–8:50
☐	Section 003	Daniel Martin	8:00–8:50
☐	Section 004	Albert Bronstein	9:00–9:50
☐	Section 005	Mark Pullins	9:00–9:50
☐	Section 006	Xingzhou Yang	9:00–9:50
☐	Section 007	Martin Walter	10:00–10:50
☐	Section 008	Kevin Manley	10:00–10:50
☐	Section 009	Albert Bronstein	1:00–1:50
☐	Section 010	Martin Walter	1:00–1:50
☐	Section 011	Xingzhou Yang	2:00–2:50
☐	Section 012	Taylor Klotz	2:00–2:50
☐	Section 013	Xingzhou Yang	3:00–3:50
☐	Section 014	Braden Balentine	4:00–4:50
☐	Section 015	Caroline Matson	4:00–4:50

Question	Points	Score
1	10	
2	6	
3	10	
4	8	
5	12	
6	12	
7	12	
8	10	
9	10	
10	10	
Total:	100	

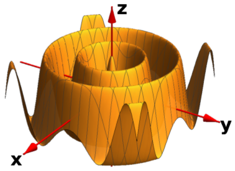
Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

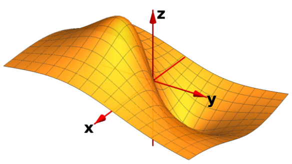
- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like $\mathbf{100/7}$ or expressions like $\mathbf{\ln(3)/2}$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (10 points) Match each 3D surface with one of the contour plots, and one of the equations.

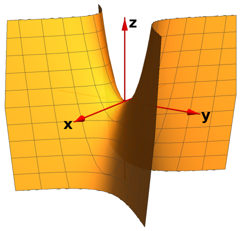
(a) ____, ____



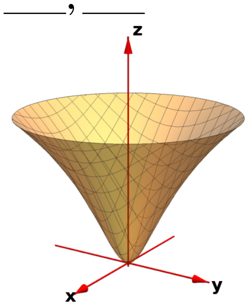
(b) ____, ____



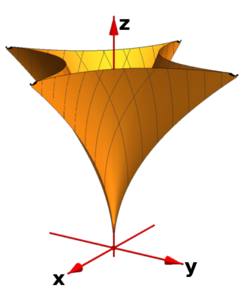
(c) ____, ____



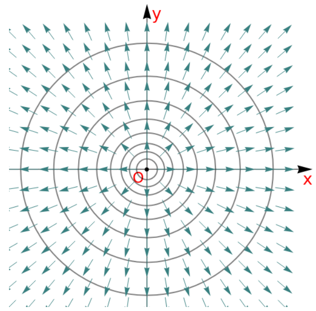
(d) ____, ____



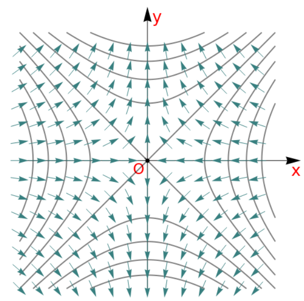
(e) ____, ____



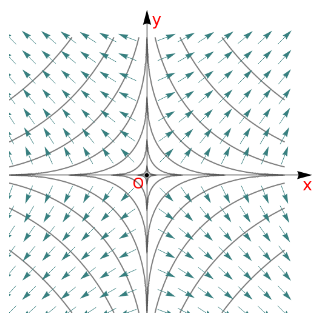
(1)



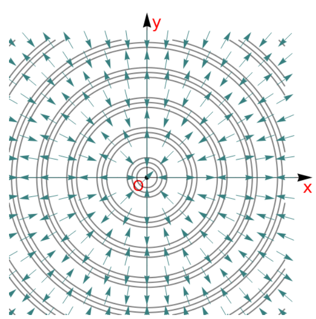
(2)



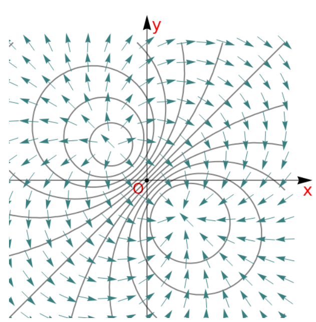
(3)



(4)



(5)



(A) $z = \dfrac{x - y}{1 + x^2 + y^2}$

(B) $z = \ln(1 + x^2 + y^2)$

(C) $z = y^2 - x^2$

(D) $z = \sqrt{|x|} + \sqrt{|y|}$

(E) $z = \cos(\sqrt{x^2 + y^2})$

2. **Note: No partial credit for this problem.**

Consider function $\boldsymbol{f(x,y) = e^x + \cos(xy) + y^7}$. Compute

(a) (2 points) $\boldsymbol{f_x(x,y) =}$ ________________________.

(b) (2 points) $\boldsymbol{f_y(x,y) =}$ ________________________.

(c) (2 points) $\boldsymbol{\nabla f(0,1) =}$ ________________________.

3. **Note: No partial credit for this problem.**

Consider the surface $\boldsymbol{S}$: $\boldsymbol{z = f(x, y) = x^3 - xy + 3}$.

(a) (5 points) The directional derivative of $\boldsymbol{f}$ at $\mathbf{(0, 1)}$ in the direction of $\vec{\boldsymbol{v}} = \left\langle \mathbf{\frac{3}{5}}, \mathbf{\frac{4}{5}} \right\rangle$ is

____________.

(b) (5 points) The equation for the tangent plane to the surface $\boldsymbol{S}$ at $\boldsymbol{P}\mathbf{(0, 1, 3)}$ is

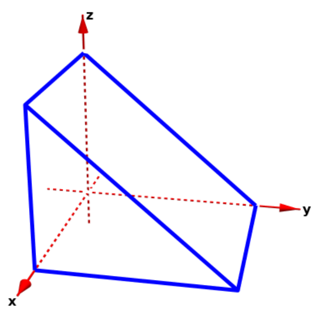
____________________________________.

4. Consider the integral $\displaystyle\int_0^1 \int_0^2 \boldsymbol{y \, dy \, dx}$

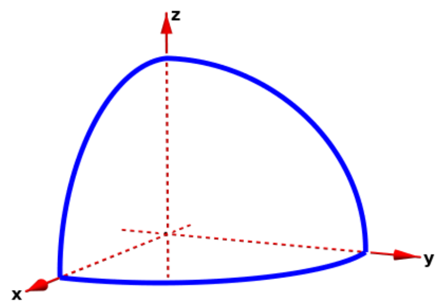
(a) (4 points) Evaluate the integral. **Note: No partial credit for this problem.**

(b) (4 points) The integral above can be interpreted as the volume of a solid. Which of the following is the best representation of the solid?

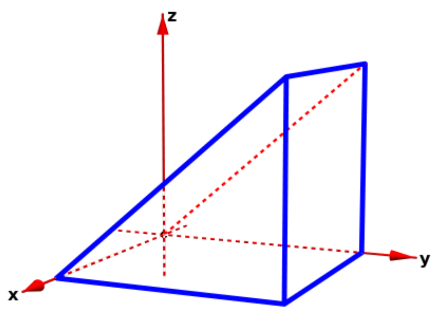
(1)



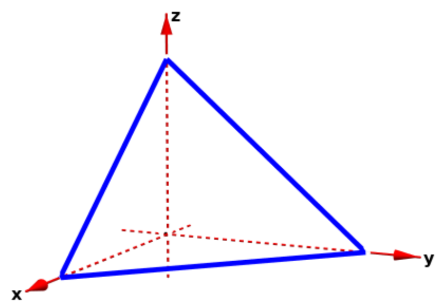
(4)



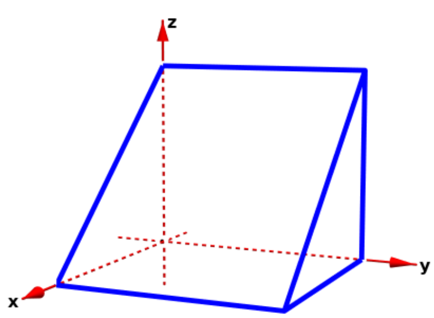
(2)



(5)

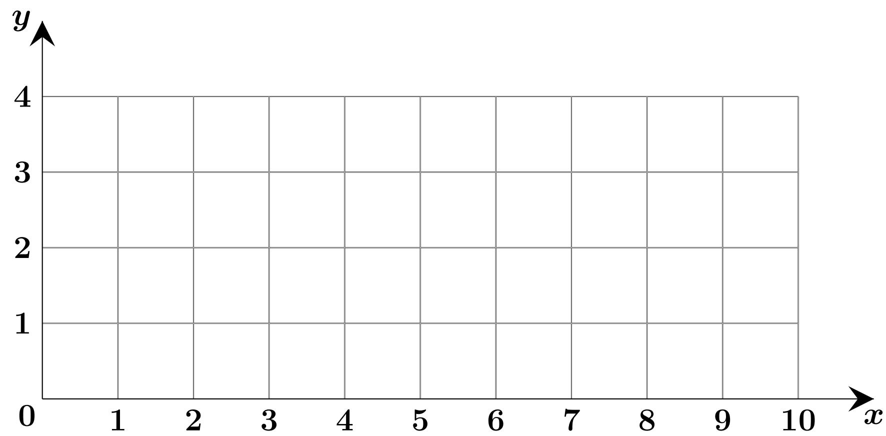


(3)



5. Consider the integral $\int_0^3 \int_{y^2}^9 y \sin(x^2)\, dx\, dy$.

(a) (4 points) Sketch the region of integration.



(b) (8 points) Evaluate the integral.

6. (12 points) Consider the function

$$f(x,y) = \begin{cases} \dfrac{y^2+x}{y^2+y\sqrt{x}+x} & (x,y)\neq(0,0) \\ 1 & (x,y)=(0,0) \end{cases}$$

Is f continuous at $(x,y)=(0,0)$? Justify your answer.

7. (12 points) Find all critical points of the function

$$\boldsymbol{f(x, y) = 2x^3 - 6xy + y^2 + 7}$$

and classify each as a local maximum, local minimum, saddle point, or not enough information. Justify your answer.

8. You are hiking up a mountain whose elevation is given roughly by

$$\boldsymbol{f(x,y) = x^2 - y^3 + xy^2 + 2.}$$

You are at $\mathbf{(1,-1,5)}$ in this model.

(a) (6 points) What is the steepest uphill direction from your position?

(b) (4 points) Suppose you wish to move from your current location while maintaining your current elevation. Find a vector parallel to the direction in which you should move.

9. (10 points) Use the **method of Lagrange multipliers** to find the extreme values of the function $\boldsymbol{f(x,y) = 2x - y^2}$ subject to the contraint $\boldsymbol{x^2 + 3y^2 = 36}$.

10. Suppose that $\boldsymbol{z} = \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{y})$ and that $\boldsymbol{x} = \boldsymbol{X}(\boldsymbol{s}, \boldsymbol{t})$ and $\boldsymbol{y} = \boldsymbol{Y}(\boldsymbol{s}, \boldsymbol{t})$, where $\boldsymbol{F}$, $\boldsymbol{X}$ and $\boldsymbol{Y}$ all have continuous partial derivatives at all points.

Suppose that the following facts are given:

$\dfrac{\partial z}{\partial x}(1, 11) = b$	$\dfrac{\partial z}{\partial x}(c, d) = 5$	$\dfrac{\partial z}{\partial y}(1, 11) = 11$	$\dfrac{\partial z}{\partial y}(c, d) = 12$
$\dfrac{\partial x}{\partial s}(c, d) = 10$	$\dfrac{\partial x}{\partial t}(c, d) = 7$	$\dfrac{\partial y}{\partial s}(c, d) = a$	$\dfrac{\partial z}{\partial t}(c, d) = 0$
$X(a, b) = -1$	$Y(a, b) = -7$	$X(c, d) = 1$	$Y(c, d) = 11$

Note: One can view $\boldsymbol{z}$ as a function of $\boldsymbol{x}$ and $\boldsymbol{y}$, and one can view $\boldsymbol{z}$ as a function of $\boldsymbol{s}$ and $\boldsymbol{t}$.

(a) (5 points) Find $\dfrac{\partial z}{\partial s}(c, d)$

(b) (5 points) Find $\dfrac{\partial y}{\partial t}(c, d)$