Math 2400, Midterm 1

September 24, 2018

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50
☐	Section 002	Philip Kopel	8:00–8:50
☐	Section 003	Daniel Martin	8:00–8:50
☐	Section 004	Albert Bronstein	9:00–9:50
☐	Section 005	Mark Pullins	9:00–9:50
☐	Section 006	Xingzhou Yang	9:00–9:50
☐	Section 007	Martin Walter	10:00–10:50
☐	Section 008	Kevin Manley	10:00–10:50
☐	Section 009	Albert Bronstein	1:00–1:50
☐	Section 010	Martin Walter	1:00–1:50
☐	Section 011	Xingzhou Yang	2:00–2:50
☐	Section 012	Taylor Klotz	2:00–2:50
☐	Section 013	Xingzhou Yang	3:00–3:50
☐	Section 014	Braden Balentine	4:00–4:50
☐	Section 015	Caroline Matson	4:00–4:50

Question	Points	Score
1	10	
2	12	
3	12	
4	4	
5	8	
6	6	
7	8	
8	10	
9	10	
10	10	
11	10	
Total:	100	

Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like $\mathbf{100/7}$ or expressions like $\mathbf{\ln(3)/2}$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (10 points) **Note: No partial credit for this problem.**
Let $\vec{\boldsymbol{a}} = \langle \mathbf{1, 2, 1} \rangle$, $\vec{\boldsymbol{b}} = \langle \mathbf{-2, 1, -3} \rangle$. Compute

(a) $|\vec{\boldsymbol{a}}| =$ ________________.

(b) $\mathbf{3}\vec{\boldsymbol{a}} - \vec{\boldsymbol{b}} =$ ________________.

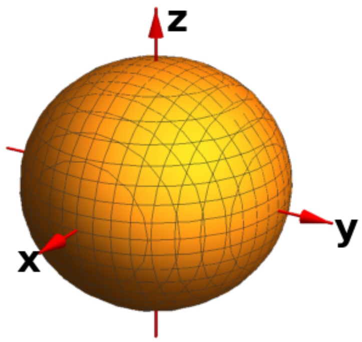
(c) $\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} =$ ________________.

(d) $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} =$ ________________.

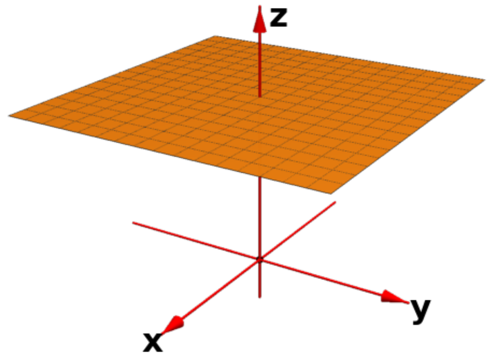
(e) $\mathrm{proj}_{\vec{\boldsymbol{a}}}\vec{\boldsymbol{b}} =$ ________________.

2. (12 points) Match each 3D surface with one of the equations. Not all equations will be matched.

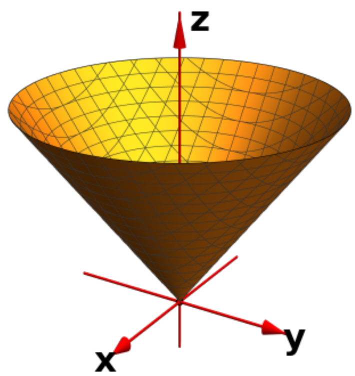
(a) ______



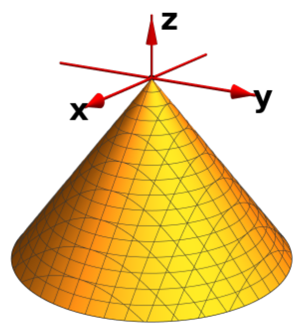
(b) ______



(c) ______



(d) ______



Let $(\boldsymbol{\rho}, \boldsymbol{\theta}, \boldsymbol{\phi})$ be spherical coordinates.

(1) $\boldsymbol{\rho = 3}$

(2) $\boldsymbol{\theta = \frac{\pi}{3}}$

(3) $\boldsymbol{\phi = \frac{2\pi}{3}}$

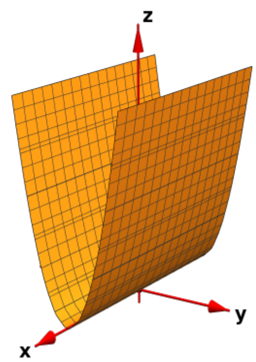
(4) $\boldsymbol{\rho = \sec\theta}$

(5) $\boldsymbol{\phi = \frac{\pi}{3}}$

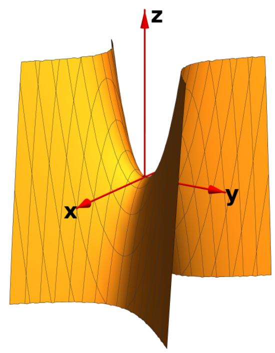
(6) $\boldsymbol{\rho = \sec\phi}$

3. (12 points) Match each 3D surface with one of the equations on the right side. Not all equations will be matched.

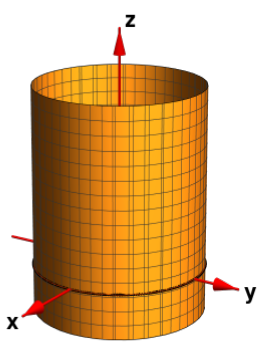
(a) ______



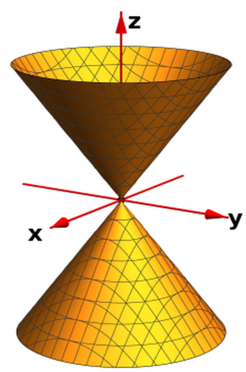
(b) ______



(c) ______



(d) ______



(1) $\boldsymbol{x^2 + y^2 - z^2 = 0}$

(2) $\boldsymbol{x^2 - y^2 + z^2 = 0}$

(3) $\boldsymbol{x^2 + y^2 - 4 = 0}$

(4) $\boldsymbol{x^2 - y^2 - 4 = 0}$

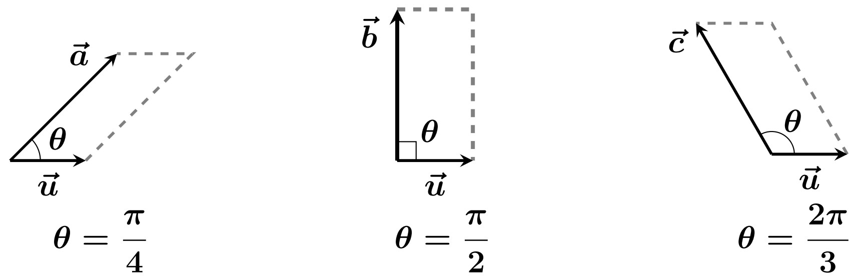
(5) $\boldsymbol{z = y^2 - x^2}$

(6) $\boldsymbol{z = x^2 - y^2}$

(7) $\boldsymbol{z = x^2}$

(8) $\boldsymbol{z = y^2}$

4. (4 points) In the pictures below assume that $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and that $\vec{u}$ is a unit vector. Use the parallelograms below to rank $|\vec{a} \times \vec{u}|, |\vec{b} \times \vec{u}|$, and $|\vec{c} \times \vec{u}|$ from smallest to largest.



Circle one of the following.

(a) $|\vec{a} \times \vec{u}| \leq |\vec{b} \times \vec{u}| \leq |\vec{c} \times \vec{u}|$

(b) $|\vec{a} \times \vec{u}| \leq |\vec{c} \times \vec{u}| \leq |\vec{b} \times \vec{u}|$

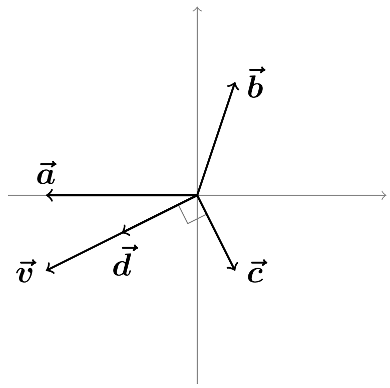
(c) $|\vec{b} \times \vec{u}| \leq |\vec{a} \times \vec{u}| \leq |\vec{c} \times \vec{u}|$

(d) $|\vec{b} \times \vec{u}| \leq |\vec{c} \times \vec{u}| \leq |\vec{a} \times \vec{u}|$

(e) $|\vec{c} \times \vec{u}| \leq |\vec{a} \times \vec{u}| \leq |\vec{b} \times \vec{u}|$

(f) $|\vec{c} \times \vec{u}| \leq |\vec{b} \times \vec{u}| \leq |\vec{a} \times \vec{u}|$

5. (8 points) Consider the following vectors $\vec{\boldsymbol{v}}$, $\vec{\boldsymbol{a}}$, $\vec{\boldsymbol{b}}$, $\vec{\boldsymbol{c}}$, and $\vec{\boldsymbol{d}}$ in the $\boldsymbol{xy}$-plane.



Circle whether each of the following is true or false.

(a) **TRUE** or **FALSE**: $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{a}} \geq \vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{d}}$

(b) **TRUE** or **FALSE**: $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{c}} = \vec{\boldsymbol{0}}$

(c) **TRUE** or **FALSE**: $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{a}} = \vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{b}}$

(d) **TRUE** or **FALSE**: $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{d}} = |\vec{\boldsymbol{v}}|^{\boldsymbol{2}}$

6. (6 points) Let $\boldsymbol{C}$: $\vec{\boldsymbol{r}}(\boldsymbol{t}) = \langle \boldsymbol{x}(\boldsymbol{t}), \boldsymbol{y}(\boldsymbol{t}), \boldsymbol{z}(\boldsymbol{t}) \rangle$ be any space curve such that $|\vec{\boldsymbol{r}}\,'(\boldsymbol{t})| = \mathbf{1}$. If $\mathbf{3} \le \boldsymbol{t} \le \mathbf{5}$ then what is the length of $\boldsymbol{C}$ from $\vec{\boldsymbol{r}}(\mathbf{3})$ to $\vec{\boldsymbol{r}}(\mathbf{5})$?

7. (8 points) Find a parametric representation of the part of the plane $\boldsymbol{z} = \boldsymbol{x} + \mathbf{2}$ that lies inside the cylinder $\boldsymbol{x}^{\mathbf{2}} + \boldsymbol{y}^{\mathbf{2}} = \mathbf{1}$.

8. (10 points) Consider the space curve $\boldsymbol{C}$: $\vec{\boldsymbol{r}}(\boldsymbol{t}) = \left\langle \boldsymbol{t} \ln \boldsymbol{t},\ \boldsymbol{2t},\ \boldsymbol{t^2} \right\rangle$, where $\boldsymbol{t} > \boldsymbol{0}$.

(a) Calculate $\dfrac{\boldsymbol{d}}{\boldsymbol{dt}}\left[\vec{\boldsymbol{r}}(\boldsymbol{t})\right]$.

(b) Find the parametric equations of the line tangent to $\boldsymbol{C}$ at the point $(\boldsymbol{0}, \boldsymbol{2}, \boldsymbol{1})$.

9. (10 points) The positions of two particles at time $\boldsymbol{t}$, for $\boldsymbol{t \geq 0}$, are given by $\vec{\boldsymbol{r}}_1(\boldsymbol{t}) = \left\langle \boldsymbol{t},\ \boldsymbol{t^2 + 2t},\ \boldsymbol{1} \right\rangle$ and $\vec{\boldsymbol{r}}_2(\boldsymbol{t}) = \left\langle \boldsymbol{2t - 1},\ \boldsymbol{4t^2 - 1},\ \boldsymbol{\cos(\pi t)} \right\rangle$, respectively. Do the two particles collide? If so, then at what time(s) do they collide? If not, justify why not.

10. (10 points) Find an equation for the plane through the point $\boldsymbol{P(2,3,0)}$ and parallel to a plane determined by vectors $\vec{\boldsymbol{u}} = \langle \mathbf{1},\ \mathbf{2},\ \mathbf{3} \rangle$ and $\vec{\boldsymbol{v}} = \langle \mathbf{3},\ \mathbf{4},\ \mathbf{5} \rangle$.

11. (10 points) Given $\boldsymbol{P(1,\ 2,\ 3)}$ and a line $\boldsymbol{L:\ \frac{x-1}{2} = \frac{y+2}{1} = \frac{z-3}{2}}$, find the shortest distance between the point $\boldsymbol{P}$ and the line $\boldsymbol{L}$.