

Math 2400, Midterm 1

June 19, 2017

PRINT YOUR NAME: _____

PRINT INSTRUCTOR'S NAME: _____

Mark your section/instructor:

<input type="checkbox"/>	Section 400	Ilia Mishev	9:15am - 10:20am
<input type="checkbox"/>	Section 401	Joseph Timmer	11:00am - 12:05pm
<input type="checkbox"/>	Section 402	Michael Roy	12:45pm - 1:50pm

Question	Points	Score
1	15	
2	16	
3	12	
4	12	
5	11	
6	12	
7	12	
8	10	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions unless otherwise stated. For example leave fractions like $100/7$ or expressions like $\ln(3)/2$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.

1. (15 points) Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be **non-zero** vectors in \mathbb{R}^3 and θ the angle between \mathbf{u} and \mathbf{v} .

(a) True or False: $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

(b) True or False: $\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u}$.

(c) True or False: $|\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w})| = |\mathbf{v} \bullet (\mathbf{w} \times \mathbf{u})|$.

(d) True or False: $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$.

(e) True or False: $\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \sin \theta$.

2. (16 points) Let $\mathbf{u} = \langle 2, 2, -1 \rangle$ and $\mathbf{v} = \langle 1, 2, 2 \rangle$. Find:

(a) $3\mathbf{u} - 2\mathbf{v}$.

(b) $\mathbf{u} \bullet \mathbf{v}$.

(c) $\mathbf{u} \times \mathbf{v}$.

(d) The angle between \mathbf{u} and \mathbf{v} .

3. (12 points) At what point(s) (x, y, z) does the curve $\mathbf{r}(t) = \langle 2t - 1, t^2 - t, t \rangle$ intersect the plane $2x + 3y - z - 1 = 0$?

4. (12 points) The position of a particle at time t is given by the position vector

$$\mathbf{r}(t) = \langle \cos(t^2 - 4t), e^{2t^3 - 3t^2 - 12t}, 2 \sin(t^2 - 4t) \rangle.$$

At what time(s) does the particle stop moving?

5. (11 points) A particle has acceleration $\mathbf{r}''(t) = \hat{\mathbf{i}} + t\hat{\mathbf{j}}$. If it has initial velocity $\mathbf{r}'(0) = 2\hat{\mathbf{k}}$ and initial position $\mathbf{r}(0) = 3\hat{\mathbf{i}} - 2\hat{\mathbf{k}}$, compute its position vector $\mathbf{r}(t)$.

6. (12 points) Find the equation of the plane P that passes through the point $(3, 0, -1)$ and contains the line with parametric equations $x = 1 - 2t, y = -2 + t, z = 5$.

7. (12 points) Let $f(x, y) = (a^2 - 2)x^2 + (2a + 1)y^2$, where a is a real number. Find the value of a for which the trace of the graph of f in the plane $z = -4$ is a circle of radius 2.

8. (10 points) Convert the equation of the cone $z = \sqrt{\frac{x^2 + y^2}{3}}$ (given here in rectangular coordinates) to spherical coordinates. Fully simplify your answer.